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21BSC2C2MTL

B.Sc. II Semester Degree Examination, September/October - 2023

MATHEMATICS

Algebra and Calculus - II

(NEP)

Time : 2 Hours | Maximum Marks : 60

Note : *Answer* ***all*** *Parts.*

PART - A

1. Answer **all** questions. **10x1=10**

(a) Define neighbourhood of a point.

(b) Define limit point of a set.

(c) Define centre of a Group.

(d) If 'a' is a generator of a cyclic group G then prove that a^{-1} is also a generator.

(e) If $u = 3x+5y$, $v = 4x-3y$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.

(f) Find the degree of the homogeneous function $u = \frac{x^{1/3} - y^{1/3}}{x^{1/2} + y^{1/2}}$.

(g) Show that $\int_C [(x + y)dx + (x - y)dy] = 0$ where C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$.

(h) Evaluate : $\int_0^1 \int_0^2 (x + y)dxdy$.

(i) Write the surface area formula (S) whose projection on *yz*-plane.

(j) Evaluate : $\int_0^2 \int_0^2 \int_0^2 dydxdz$

PART - B

Answer **any four** of the following. **4x5=20**

2. The union of a finite number of closed sets is a closed set.

3. State and prove Lagrange's theorem.

P.T.O.

4. If $u = \sin^{-1}\left(\frac{x^3 - y^3}{x - y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$

5. Evaluate $\int_C [(x+y)dx+(y-x)dy]$ along the parabola $y^2 = x$ from (1, 1) to (4, 2).

6. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$

7. Evaluate : $\int\int_A xy\,dx\,dy$, where A is the region bounded by the co-ordinate axes and the line $x+y = 1$.

PART - C

Answer **any three** of the following questions. **3x10=30**

8. (a) Find the supremum and infimum of S = {2, 4, 6, 10, 12}.

(b) State and prove Archimidean property of R.

9. (a) If H and K are any two subgroups of a group G then prove that HK is a subgroup of G iff HK = KH.

(b) Show that every factor group of a cyclic group is cyclic.

10. (a) If $u = f(x, y)$ be a homogeneous function of degree n^{-1} then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 x}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

(b) Find $\frac{dz}{dt}$, if $z = x^2 + y^2$, where $x = e^t\cos t$, $y = e^t\sin t$.

11. (a) Show that $\int_C y^2dx + 2xy\,dy$ is independent of the path joining (0, 1) and (1, 3) and hence evaluate.

(b) Change the order of integration and evaluate $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y}dy\,dx$.

12. (a) Evaluate : $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}$

(b) Find the volume of the sphere $x^2 + y^2 + z^2 = 2^2$.

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