

**B.Sc. II Semester (NEP) Degree Examination, September/October - 2022****MATHEMATICS****Algebra and Calculus - II**

Time : 3 Hours

Maximum Marks : 60

**Instruction :** Answer **all** Parts.**PART - A****1.** Answer **all** questions.**10x1=10**

(a) Define Closed and Open interval of a set.

(b) Define Neighbourhood of a point.

(c) If  $a, b$  are any two elements of a group  $G$  then  $(ab)^2 = a^2b^2$  if and only if  $G$  is abelian.(d) If ' $a$ ' is a generator of a cyclic group  $G$  then prove that  $a^{-1}$  is also generator of  $G$ .(e) If  $x = r \cos\theta$  and  $y = r \sin\theta$  show that  $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$ .(f) If  $u = 3x + 5y$ ,  $v = 4x - 3y$  then find  $\frac{\partial(u, v)}{\partial(x, y)}$ .(g) Evaluate  $\int_0^1 \int_1^2 (x^2 + y^2) dy \cdot dx$ .(h) Evaluate  $\int_C [(3x + y)dx + (2y - x)dy]$ , along  $y = x^2 + 1$  from  $(0, 1)$  and  $(3, 10)$ .(i) Evaluate  $\int_C [y^2 \cdot dx + 2xy \cdot dy]$  along  $C$  from  $(0, 0)$  to  $(1, 3)$  by checking it for path independence.(j) Evaluate  $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{6}} \sin x \cdot \cos y \cdot dx \cdot dy$ .

## PART - B

Answer **any four** of the following questions.

**4x5=20**

2. Show that union of two open sets is an open set.
3. Show that a subset  $H$  of a group  $G$  is a sub-group of  $G$  if and only if  $H H^{-1} = H$ .
4. If  $u = f(x, y)$  is a homogenous function of degree  $n$  then show that
 
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$
5. If  $u$  and  $v$  are functions of two independent variables  $s$  and  $t$  and  $s$  and  $t$  themselves are functions of two independent variables  $x$  and  $y$ , then
 
$$\frac{\partial(u,v)}{\partial(s,t)} \cdot \frac{\partial(s,t)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$
6. Evaluate  $\int_C \left( \frac{a^2 y^2}{b^2} + \frac{b^2 x^2}{a^2} \right) ds$  around the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
7. Evaluate  $\iiint_R (x+y+z) dx \cdot dy \cdot dz$  where  $R$  is the region bounded by the planes  $x=0, y=0, z=0$  and  $x+y+z=1$ .

## PART - C

Answer **any three** of the following questions.

**3x10=30**

8. (a) State and prove Lagrange's theorem.  
 (b) Define sub-group of a group and show that if  $H$  is a sub-group of  $G$  then  $H^{-1} = H$ .
9. (a) Define Supremum of a set. Show that the supremum of a non-empty set  $S$  of real numbers, whenever it exists, is unique.  
 (b) State and prove Archimedean property of  $\mathbb{R}$ .



10. (a) Show that the functions  $u = x + y + z$ ,  $v = x - y + z$ ,  $w = x^2 + y^2 + z^2 - 2yz$  are functionally dependent and find the relation connecting them.
- (b) Obtain Taylor's Series expansion of  $f(x, y) = x^2 + y^2 - xy$  in the powers of  $(1, -2)$ .

11. (a) Evaluate  $\int_C \left[ \frac{x}{x^2 + y^2} \cdot dy - \frac{y}{x^2 + y^2} \cdot dx \right]$  around C where C is the circle  $x^2 + y^2 = 1$

traversed once in the anti-clockwise direction.

- (b) Define Criterion for line integral to be independent of path and evaluate

$$\int_C [y^2 \cdot dx + 2xy \cdot dy]$$

along C from  $(0, 0)$  to  $(1, 3)$  by checking it for path independence.

12. (a) Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \cdot dy \cdot dz$

- (b) Find the volume of the sphere  $x^2 + y^2 + z^2 = a^2$

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