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Sl. No.

21MAT1C1L

M.Sc. I Semester Degree Examination, April/May - 2023

MATHEMATICS

Algebra

Time : 3 Hours | Maximum Marks : 70

Note : *Answer **any five** questions with **Q.1 compulsory.** **All** questions carries **equal** marks.*

1. (a) Show that any two cyclic groups of same order are isomorphic. **6+6+2**

(b) If $G = <a>$ is a cyclic group generated by an element $a \epsilon G$. Then prove the following :

(i) If O(a) is zero then G is isomorphic additive group of integers.

(ii) If O(a) is n>O, then G is isomorphic to residue class modulo n.

(c) Express the following permutation as a product of disjoint cycles.

(1 2 3) (4 5) (1 3 4 5)

2. (a) State and prove Fundamental theorem of group isomorphism. **5+5+4**

(b) State and prove Sylow's 1^{st} Theorem.

(c) Give an example of non-abelian group G and proper normal subgroup N of G such that $\frac{G}{N}$ is abelian.

3. (a) Let G be a group C(G) is its centre and I(G) be the group of inner Automorphism then show that $\frac{G}{C(G)} \cong I(G)$. **5+5+4**

(b) State and prove Cauchy's theorem of abelian group.

(c) Show that there are only two groups of order 6 one is cyclic and other is isomorphic to S_3.

4. (a) Show that H is maximal subgroup of G if and only if $\frac{G}{H}$ is simple. **6+4+4**

(b) Show that S_4 is solvable group.

(c) If $O(G) = 56$, then prove that G has 1 or 8 sylow 7 subgroup(s).

P.T.O.

5. (a) Show that a non-zero commutative ring with unity is a field if it has no-proper ideal. **6+6+2**

(b) If $M_2(R)$, the set of all matrices of 2×2 order over R is non-commutative ring with unity under addition and multiplication of matrices. Prove that $M_2(R)$ is a regular ring.

(c) Define Sylow's p-subgroup and give an example.

6. (a) Let G be a group $a\epsilon G$, define a mapping $\phi_a : G\to G$ by $\phi_a(g) = a^{-1}ga$ then show that ϕ_a is automorphism. **6+4+4**

(b) Show that the ring of Gaussian integer $z[i]$ is an Euclidean domain.

(c) Prove that $z\sqrt{2} = \left\{a + \frac{\sqrt{2}b}{a}, b \epsilon z\right\}$ is a Euclidean domain.

7. (a) Let R be an integral domain then show that $\exists$ an embeding of R into a field.

(b) Prove that every ideal in a Euclidean domain is principal ideal. **6+4+4**

(c) If R is a system with 1. Satisfying all axiom of a ring except possible $a+b=b+a$ for a, $b\epsilon R$, then show that R is a ring.

8. (a) Define unique Factorization Domain with example. **4+5+5**

(b) State and prove Gauss Lemma.

(c) Show that $f(x) = 8x^3 - 2x^2 - 5x + 10$ is irreducible over Q.

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