No. of Printed Pages : **2**

Sl. No.

21MAT1C3L

M.Sc. I Semester Degree Examination, April/May - 2023

MATHEMATICS

Differential Equations

Time : 3 Hours   Maximum Marks : 70

Note : *Answer **any five** questions with Question Number **1** is **compulsory**. **All** questions carries **equal** marks.*

1. (a) State and prove Liouville's theorem. **7**

(b) Find the Wronskian of the independent solution $y^{V}-y^{IV}-y^{I}+y=0$ in [0,1] **7**

2. (a) Solve by the method of undetermined coefficients, $y''+2y'+y=2\cos x-3x+2+3e^{x}$. **5**

(b) Define : **5**

(i) Adjoint differential equation

(ii) Self adjoint differential equation

(iii) Normalized differential equation

(iv) The wronskian

(v) Linear dependence and independence

(c) Is Hermite differential equation is self adjoint ? If not transform it as an equivalent self-adjoint form. **4**

3. (a) Define oscillatory and non-oscillatory differential equations with an example. **5**

(b) State and prove Sturm-comparision theorem. **4**

(c) Show that the differential equation $y''+\frac{k}{x^2}y=0$ $(-\infty\leq x<\infty)$ where k is constant and x>0 is oscillatory, if $k>\frac{1}{4}$ and non-oscillatory if $k\leq\frac{1}{4}$. **5**

4. (a) Find the general solution of $x^2y'' + 9xy' + 12y = 0$ by finding the solution of its adjoint equation. **5**

(b) Define the following : **5**

(i) Orthogonality.

(ii) Orthogonal set of functions.

(iii) Orthonormal set of functions.

(iv) Orthogonality w.r.t. a weight function.

(v) Orthogonal set of functions with respect to a weight function.

(c) Show that the set of functions. **4**

(i) $\left\{\sin \frac{n\pi x}{c}\right\}$ $n = 1, 2, 3, \ldots\ldots$ is orthogonal on the interval (0,c).

(ii) $\{\cos nx\}$ $n = 0, 1, 2, 3, \ldots\ldots$ is orthogonal on the interval $-\pi \leq x \leq \pi$. Hence find the orthonormal set.

5. (a) If a power series $\sum a_n x^n$ converges for $x = x_0$, then prove that : **4**

(i) It is absolutely convergent in the interval $|x| < |x_0|$

(ii) It is uniformly convergent in the interval $|x| \leq |x_1|$, where $|x_1| < |x_0|$

(b) Find the power series solution of the equation $y'' + xy' + x^2y = 0$ about origin. **5**

(c) Find the solution near $x = 0$ of $x^2y'' + (x + x^2)y' + (x - 9)y = 0$, by Frobenius method. **5**

6. (a) Find the fundamental matrix solution of the following system of equations : **7**

$$\frac{dx}{dt} = 4x - y; \quad \frac{dy}{dt} = x + 2y.$$

(b) Determine the critical points of the system $\frac{dx}{dt} = x + y; \ \frac{dy}{dt} = 3x - y.$ **7**

Discuss the nature and stability of the critical point and obtain the general solution of the system.

7. (a) Find the Eigen value and Eigen function of $y'' + \lambda y = 0;\ y(0) = y(\pi) = 0$. **7**

(b) Solve by the method of variation of parameters $x^2y'' - 2y = x^3$ and $y_1 = x^2$ is a solution of homogeneous equation. **7**

8. (a) Solve the Bessel's equation near zero $xy'' + y' + xy = 0$ in series by Frobenius method. **5**

(b) Apply Liapunor direct method to determine the stability of the critical point (0, 0) of the following system. **5**

(i) $\frac{dx}{dt} = -y + x^3; \ \frac{dy}{dt} = x + y^3$

(ii) $\frac{dx}{dt} = y - 2x^3; \ \frac{dy}{dt} = -2x - 3y^5$

(c) State and prove Greens formula. **4**

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