No. of Printed Pages : **2**

Sl. No.

21MAT1C5L

M.Sc. I Semester Degree Examination, April/May - 2023

MATHEMATICS

Topology

Time : 3 Hours   Maximum Marks : 70

***Note :** Answer **any five** questions with question **No.1 Compulsory.** **All** questions carry **equal** marks.*

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1. (a) Let (X, d) be a metric space and let $< F_n >$ be a sequence of non-empty closed subsets of X such that $\delta(F_n) \to 0$ as $n \to \infty$. Then show that X is complete if and only if $\bigcap_{n=1}^{\infty} F_n$ consists of exactly one point.

(b) Show that closed open interval [a, b) is neither closed nor open for the usual metric on R. **(10+4)**

2. (a) If (X, τ) is a topological space and $A \subset x$ then prove the following :

(i) $(X - A)^{\circ} = \left(X - \overline{A}\right)$

(ii) $(X - A^{\circ}) = \left(\overline{X - A}\right)$

(iii) $b\left(A^{\circ}\right) \subset b(A)$

(iv) $\overline{A} = A^{\circ} \cup b\,(A)$

(b) Find the interior, closure, boundary set and derived set of the following subsets of R relative to usual topology.

(i) (0, 1)

(ii) Q, the set of irrational numbers

(iii) N, the set of all natural numbers **(8+6)**

3. (a) Let X and Y be two topological spaces then show that a map $f : X \to Y$ is continuous if and only if for every closed set $F \subset y$, f^{-1} (F) is closed set in X.

(b) Give an example to show that every open cover of second axiom space may not reducible to a countable subcover. **(7+7)**

P.T.O.

4. State and prove Urysohn's lemma. **(14)**

5. (a) If (X, τ) is a topological space and A is a non-empty subset of X then show that A is disconnected if and only if A is the union of two non-empty separated sets.

(b) Show that a topological X is compact if and only if every collection of closed subsets of X with the finite intersection property is fixed. **(7+7)**

6. (a) let (X, τ) be a topological space and $A \subset X$ then prove that $A \cup D(A)$ is closed and $\overline{A} = A \cup D(A)$.

(b) Define open map. Let X and Y be two topological spaces, then show that a map $f : X \to Y$ is an open map if and only if for every $A \subset X$, $f(A^{\circ}) \subset (f(A))^{\circ}$. **(7+7)**

7. (a) Let (X, τ) be a T_1-space and $A \subset X$. If a point $x \in X$ is a cluster point of A then show that every neighbourhood of x contains infinitely many points of A.

(b) Prove that continuous image of a connected space is connected set. **(7+7)**

8. (a) Show that in a topological space (X, τ) arbitrary intersection of closed sets is closed and finite union of closed sets is closed.

(b) Show that the topological space (R, U) and (R, I) are not homeomorphic, where U and I are usual topology and indiscrete topology on R.

(c) Prove the following :

(i) Every cofinite space is a T_1- space.

(ii) Every T_4- space is a T_3- space. **(5+5+4)**

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