No. of Printed Pages : **2**

21MAT2C10L

Sl. No.

M.Sc. II Semester Degree Examination, October - 2023

MATHEMATICS

Complex Analysis

(NEP)

Time : 3 Hours Maximum Marks : 70

Note : *Answer **any five** questions with question no. **1** is **compulsory**.*

1. (a) State and prove Cauchy Riemann's Equations in Cartesian form. **5+4+5**

(b) State and prove fundamental theorem of algebra.

(c) Find the bilinear transformation which maps the points $z=1,\ i,\ -1$ on the points $w=i,\ 0,\ -i$. Hence find the image of $|z|<1$ and the invariants of the transformation.

2. (a) State and prove Cauchy-Hadamard Theorem. **6+4+4**

(b) Find the radius of convergence of the power series $f(z)=\sum_{n=0}^{\infty}\frac{z^n}{2^n+1}$ and prove that $(2-z)\,f(z)-2 \to 0$ as $z \to 2$.

(c) Expand $f(z)=\frac{1}{z\left(z^2-3z+2\right)}$ for the regions.

(i) $0<|z|<1$ (ii) $|z|>2$

3. (a) Prove that a rational function has no singularities other than poles. **5+5+4**

(b) State and prove Weierstrass Theorem on Singularity.

(c) Discuss the nature of Singularities of the following functions :

(i) $f(z)=\frac{1-e^z}{1+e^z}$ at $z=\infty$

(ii) $f(z)=\sin\frac{1}{z}$ at $z=0$

P.T.O.

4. (a) State and prove Cauchy's integral theorem for simply connected region. **4+4+6**

(b) Evaluate $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2 \cdot (z-1)} dz$, where C is the circle $|z| = 3$.

(c) State and prove Schwarz lemma for analytic function.

5. (a) State and prove Phragman and Lindeloff Theorem. **2+7+5**

(b) State and prove Schwarz theorem.

(c) Use Rouche's theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$.

6. (a) Obtain the Taylor's and Laurent's series for the function. **7+7**

$f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in the regions.

(i) $|z| < 2$, (ii) $2 < |z| < 3$, (iii) $|z| > 3$

(b) State and prove Cauchy residue theorem.

7. (a) State and prove Rouche's theorem. **7+7**

(b) Evaluate $\int_C \frac{1}{z(z-1)} dz$ where C is the circle $|z| = 3$, by using Cauchy integral formula.

8. (a) State and prove Liouville's theorem for an integral function. **5+5+4**

(b) Find the residues of $\frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$ at all its poles lie in the finite plane.

(c) Show that a function which has no Singularities in the finite parts of the plane and has a pole of order 'n' at infinity is a polynomial of degree n.

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