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M.Sc. II Semester Degree Examination, September/October - 2022

MATHEMATICS

DSC 10 : 21MAT2C10L : Complex Analysis

Time : 3 Hours | Maximum Marks : 70

Note : *Answer **any five** of the following questions with Question No.**1 Compulsory**, each question carries **equal** marks.*

1. (a) State and prove Cauchy integral formula. **5**

(b) Prove that two functions u(x, y) and v(x, y) are harmonic conjugates of each other if and only if they are constants. **5**

(c) Evaluate $\int_C \frac{1}{z(z-1)}\,dz$ where C is the circle $|z| = 3$. **4**

2. (a) Prove that the sum function $f(z)$ of the power series $\sum a_n z^n = f(z)$ represents an analytic function inside its circle of convergence. **6**

(b) Find the domains of convergence of the following series : **8**

(i) $\sum_{n=1}^{\infty} \frac{1.3.5....(2n-1)}{n!}\left(\frac{1-z}{z}\right)^n$

(ii) $\sum n^2 \left(\frac{z^2+1}{1+i}\right)^n$

3. (a) State and prove Weistrass theorem. **6**

(b) Discuss the nature of singularities of the following functions : **8**

(i) $\frac{\cot \pi z}{(z-a)^2}$ at $z = 0, z = \infty$

(ii) $\sin \frac{1}{1-z}$ at $z = 1$

(iii) $\sin z - \cos z$ at $z = \infty$

(iv) $\tan \frac{1}{z}$ at $z = 0$

4. (a) If $f(z)$ is meromorphic function inside a closed contour C and has no zero on C, then prove that $\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz = N - P$ where N is the number of zero's and P is the number of poles inside C. **7**

(b) By the method of contour integration prove that **7**

$$\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta} d\theta = \frac{2\pi}{b^2}\left\{a - \sqrt{a^2 - b^2}\right\}$$ where a > b > 0.

5. (a) State and prove Schwartz's theorem. **7**

(b) State and prove Hadmard three circle theorem. **7**

6. (a) State and prove Taylor's theorem for an analytic function. **7**

(b) Use Cauchy Residue theorem to evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle : **7**

(i) $|z| = 1$

(ii) $|z + 1 - i| = 2$

(iii) $|z + 1 + i| = 2$

7. (a) State and prove Maximum Modulus theorem. **6**

(b) If a > e then prove that $e^z = az^n$ has n roots inside the circle $|z| = 1$ by using Rouche's theorem. **6**

(c) Define convex function and harmonic function. **2**

8. (a) Obtain the Taylor's or Laurent series for the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when (i) $|z| < 1$ (ii) $1 < |z| < 2$ **5**

(b) State and prove Cauchy Residue theorem. **5**

(c) State and prove Rouche's theorem. **4**

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