

**M.Sc. II Semester Degree Examination, October - 2023****MATHEMATICS****Fluid Mechanics****(NEP)**

Time : 3 Hours

Maximum Marks : 70

Note : Answer *any five* questions with question no. 1 is **Compulsory**.

1. (a) The velocity components for two-dimensional fluid system can be given in **7+7** the eulerian system by $u=2x+2y+3t$, $v=x+y+t/2$. Find the displacement of the fluid particle in the Lagrangian system.
- (b) Derive with usual notation, the equation of continuity in Cartesian coordinates.
2. (a) Derive with usual notation, the equation of motion under impulsive force in **6+8** Cartesian form and prove that impulse satisfy the Laplace equation.
- (b) Explain the followings :
- Stream line
 - Path line
 - Streak line
 - Velocity potential
3. (a) An infinite mass of fluid is acted on by a force $\frac{\mu}{r^{3/2}}$ per unit mass directed to **7+7** the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = C$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{1/2} C^{5/4}$
- (b) Deduce the Helmholtz vorticity equations.
4. (a) Define stream function. Write the physical significance of stream function. **7+7**
- (b) State and prove Bernoulli's theorem.
5. (a) Derive the expression for complex potential due to a doublet in two dimension.
- (b) What arrangement of sources and sinks will give rise to the function **5+5+4**
- $$w = \log \left(z - \frac{a^2}{z} \right).$$
- Draw a rough sketch of the streamlines. Prove that two of the streamlines subdivided into the circle.
- (c) Define image. Discuss the advantages of images in fluid dynamics.



6. (a) Discuss the application of Bernoulli's equation in trajectory of a free jet. **7+7**
(b) Derive the equation of continuity in cylindrical form.

7. (a) Find the streamlines and path lines of the particles when the velocity components are given as $u = \frac{x}{1+t}$, $v = \frac{y}{1+t}$, $w = \frac{z}{1+t}$ **4+5+5**

- (b) Derive with usual notations, Euler's equation of motion in Cartesian form.
(c) Derive Lagranges hydrodynamical equation.

8. (a) A sphere of radius R whose center is at rest vibrate radially in an infinite fluid of density 'ρ' which is at rest at infinity. If the pressure at infinity is π, show that the pressure at the surface of the sphere at time t is

$$\pi + \frac{\rho}{2} \left[\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt} \right)^2 \right] \quad \mathbf{5+4+5}$$

- (b) Derive Cauchy-Riemann equation in polar form.
(c) State and prove Kelvius circulation theorem.

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