No. of Printed Pages : **2**

21MAT2C9L

Sl. No.

M.Sc. II Semester Degree Examination, October - 2023

MATHEMATICS

Differential Geometry

(NEP)

Time : 3 Hours Maximum Marks : 70

Note : *Answer **any five** questions with question No. **1 compulsory**.*

1. (a) Define tangent vector and tangent space on E^3 and show that tangent space $T_p[E^3]$ is isomorphic to the space E^3.

(b) For any three 1 - forms $\phi_i = \sum_j f_{ij}\, dx_j \ (1 \le i \le 3)$ prove that

$$\phi_1 \wedge \phi_2 \wedge \phi_3 = \begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix} dx_1 dx_2 dx_3$$

(c) Find the unique curve $\alpha(t)$ such that $\alpha(0) = (1, 0, -5)$ and $\alpha'(t) = (t^2, t, e^t)$. **5+5+4**

2. (a) If α is a regular curve in E^3 then there exist a reparametization β of α such that β has unit speed.

(b) Compute the Frenet formulae T, N, B, the curvature and Torsion functions of the unit speed helix $\beta(s) = \left(a\cos\left(\frac{s}{c}\right), a\sin\left(\frac{s}{c}\right), b\left(\frac{s}{c}\right)\right)$ where $c = (a^2 + b^2)^{1/2}$ and a, b > 0.

(c) Define isometry. If $C : E^3 \to E^3$ is an orthogonal transformation then prove that C is an isometry of E^3. **5+5+4**

3. (a) Define Co-ordinate patch. If g is a differentiable real valued function on E^3 and C is a number then show that the subset. M : g $(x, y, z) = C$ of E^3 is a surface if the differential dg is not zero at any point.

(b) Define surface on E^3. Prove that every cylinder in E^3 is a surface in E^3. **7+7**

P.T.O.

4. (a) For the following surfaces, find the quadratic approximation near the origin.

(i) $z = \exp(x^2 + y^2) - 1$

(ii) $z = \log(\cos x) - \log(\cos y)$

(iii) $z = (x + 3y)^3$

(iv) $z = (2x + y)^2 + \exp(x^2 + y^2)$

(b) Define Normal curvature and Normal vector field. If α is a curve in $M \subset E^3$ and S is the shape operator derived from unit normal vector field U, then show $\alpha''.U = S(\alpha').\alpha'$. **8+6**

5. (a) If θ_1 and θ_2 be the dual 1 - forms on E_1 and E_2 on $M \subset E^3$. If ϕ is a 1 - form and μ is a 2 - form then prove that.

(i) $\phi = \phi(E_1).\theta_1 + \phi(E_2).\theta_2$

(ii) $\mu = \mu(E_1, E_2)\, \theta_1 \wedge \theta_2$

(b) If α be a unit speed curve in $M \subset E^3$. E_1, E_2, E_3 is an adapted frame field such that E_1 is restricted to its unit tangent T, then show that α is a geodesic of M if and only if $W_{12}(T) = 0$. **8+6**

6. (a) If F is an isometry of E^3 such that $F(0) = 0$ then show that F is an orthogonal transformation.

(b) Define regular mapping. Prove that a mapping $X : D \to E^3$ is regular if and only if $X_u(d)$ and $X_v(d)$ are u, v partial derivatives of $X(u, v) = X(d)$ are linearly independent $\forall\, d \in D$ where $D \subset E^2$. **7+7**

7. (a) Determine the principal curvature of circular cylinder in E^3.

(b) Compute the dual 1 - form, connection form W_{12} and Gaussian Curvature of the associated frame fields of the following orthogonal patches.

(i) $X(u, v) = (u\cos v, u\sin v, bv)$

(ii) $Y(u, v) = (u\cos v, u\sin v, au)$ **8+6**

8. (a) If F is an isometry of E^3, then show that there exist a unique translation T and a unique orthogonal transformation C such that F = TC or F = ToC.

(b) Define simple surface, prove that a plane in E^3 is a simple surface.

(c) Define the shape operator of $M \subset E^3$. For Each point 'P' of $M \subset E^3$, prove that the Shape operator $S : T_P(M) \to T_P(M)$ is a linear operator on the tangent plane of M at P. **5+5+4**

- o 0 o -