No. of Printed Pages : **2**

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M.Sc. II Semester Degree Examination, September/October - 2022

MATHEMATICS

21MAT2C9L DSC9 : Differential Geometry

Time : 3 Hours | Maximum Marks : 70

Note : *Answer **any five** of the following questions. Question No.**1** is **Compulsory**. Each question carries **equal** marks.*

1. (a) Define reparametrization of a curve. **5+5+4**

Deduce from the lemma

$\alpha'(t)[f]=\frac{d}{dt}(f_o\alpha)(t)$ that in the definition of directional derivative, the straight line $t \to p+tV$ may be replaced by any curve α with initial velocity Vp ie $\alpha(0) = p$ and $\alpha'(0)=Vp$.

(b) If F is a mapping from E^n to E^m. Then prove that at each point p of E^n, the derivative map $F_P : T_p(E^n) \to T_{F(p)}(E^m)$ is a linear transformation.

(c) Show that curves given by $(t, 1+t^2, t)$, (sin t, cos t, t) and (sin ht, cos ht, t) all have the same initial velocity Vp. If $f = x^2-y^2+z^2$ then compute Vp[f] by evaluating f on each of this curve.

2. (a) If $\beta : I \to E^3$ is a unit speed curve with curvature k>0 and the torsion τ, then prove that $T'=kN$, $N'=-kT+\tau B$, $B'=-\tau N$ where T, N, B are tangent, principal normal and binormal vector field. **8+6**

(b) Let β be a unit speed curve in E^3 with k>0. Then show that β is a plane curve if and only if $\tau=0$.

3. (a) Show that a plane in E^3 is a simple surface in E^3. **6+8**

(b) Show that a mapping $X : D\to E^3$ is regular if and only if $X_u(d)$ and $X_v(d)$, u, v partial derivatives of $X(u, v)=X(d)$ are linearly independent $\forall d\epsilon D$ where $D\subset E^2$.

4. (a) If α is a curve in $M\subset E^3$ then show that $\alpha''.U=S(\alpha').\alpha'$ **7+7**

(b) For each point p of $M\subset E^3$, show that the shape operator $Sp : Tp(M) \to Tp(M)$ is linear on the tangent plane of M at p.

5. (a) Let θ_1 and θ_2 be the dual 1- forms of E_1, E_2 on M. If ϕ is a 1 - form and μ is a 2-form then prove the following. **6+8**

(i) $\phi = \phi\,(E_1)\,\theta_1 + \phi\,(E_2)\theta_2$

(ii) $\mu = \mu\,(E_1, E_2)\,\theta_1 \wedge \theta_2$

(b) If p is a nonumbilic point of $M \subset R^3$ then prove that there exists a principal frame field on some neighborhood of p in M.

6. (a) Prove the following : **7+7**

(i) If S and T are translations, then ST = TS is also a translation.

(ii) If T is translation by 'a' then T has an inverse T^{-1} which is translation by '$-$a'.

(iii) Given any two points p and q of E^3, there exists a unique translation T such that $T(p) = q$.

(b) If g is a differentiable real valued function on E^3 and C is a number, then show that the subset M : g (x, y, z)= C of E^3 is a surface if the differential dg is not zero at any point of M.

7. (a) Discuss the shape operators of sphere and circular cylinder in E^3. **7+7**

(b) Prove the following :

(i) $W_{13} \Lambda W_{23} = k\theta_1 \,\Lambda\, \theta_2$

(ii) $W_{13}\,\Lambda\,\theta_2 + \theta_1 \Lambda\, W_{23} = 2H\theta_1, \Lambda\theta_2$

8. (a) If F is an isometry of E^3 then prove that there exists a unique translation T and a unique orthogonal transformation C such that $F = T_o C$. **5+5+4**

(b) Prove that any surface of revolution is a surface in E^3.

(c) If S is the shape operator gotten from E_3, where E_1, E_2, E_3 is an adapted frame field on $M \subset R^3$, then prove that for each tangent vector v to M at p.

$S(v) = W_{13}(v)\,E_1(p) + W_{23}(v)\,E_2(p)$

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