No. of Printed Pages : **2**

Sl. No.

21MAT3C12L

M.Sc. III Semester Degree Examination, April/May - 2023

MATHEMATICS

Partial Differential Equations

Time : 3 Hours | Maximum Marks : 70

Note : *Answer* ***any five*** *questions with question* ***no. 1*** *is* ***compulsory****.* ***All*** *question carry* ***equal*** *marks.*

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1. (a) Derive the solution for solving PDE of order one but of any degree (other than one) by charpits method. **7+7**

(b) Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ which is homogeneous in x, y and z.

2. (a) Solve the linear PDE : **5+5+4**

$xzp + yzq = xy$

(b) Find the complete integral of $(p^2 + q^2)x = pz$

(c) Find the integral surface of the PDE $(x - y)p + (y - x - z)q = z$ through the circle $z = 1;\ x^2 + y^2 = 1$

3. (a) Solve : $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = \cos mx.\cos ny$ **5+5+4**

(b) Reduce : $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ to its canonical form

(c) Reduce the equation

$y^2 \dfrac{\partial^2 z}{\partial x^2} - 2xy \dfrac{\partial^2 z}{\partial x \cdot \partial y} + x^2 \dfrac{\partial^2 z}{\partial y^2} = \dfrac{y^2}{x} \dfrac{\partial z}{\partial x} + \dfrac{x^2}{y} \dfrac{\partial z}{\partial y}$ to its canonical form and hence solve it.

4. (a) Solve the differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation. Subject to the following conditions. **7+7**

(i) u is infinite for $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for $x=0$ and $x=l$.

(iii) $u = lx - x^2$ for $t=0$ between $x=0$ and $x=l$.

(b) Solve the PDE by Fourier transform method :

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2};$$ if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $u(x, t)$ is bounded where $x>0$, $t>0$.

5. (a) Derive the solution of diffusion equation in cylindrical Co-ordinates. **7+7**

(b) Derive the periodic solution of one-dimensional wave equation in spherical co-ordinates.

6. (a) Describe Monge's method of integrating second order non-linear PDE in its usual notation. **7+7**

(b) Solve :

$(x-y)(xr - xs - ys + yt) = (x+y)(p-q)$

7. (a) By using the method of seperation of variables, discuss the Neumann problem for a rectangle. **7+7**

(b) An infinite string is initially at rest and that the initial displacement is $f(x)$. Determine the displacement $y(x)$ of the string by Fourier transform method.

8. (a) Solve $(D^2 + 2DD' + D'^2)\, z = e^{2x+2y}$ **4+5+5**

(b) Solve $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ by D' Alembert's solution of the wave equation.

(c) Solve by Monge's method

$y^2 r + 2xys + x^2 t + px + qy = 0$

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