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Sl. No.

21MAT3E1BL

M.Sc. III Semester Degree Examination, April/May - 2023

MATHEMATICS

DSE1 (B) : Matrix Computations

Time : 3 Hours | Maximum Marks : 70

Note : *Answer **any five** question with **Q (1) compulsory**, each question carry **equal** marks.*

1. (a) Let $A \epsilon \mathbf{R}^{m \times n}$. Define row space and nullspace of A. Report the orthogondity relationship between $C(A^T)$ and N(A). **5**

(b) Let $A \epsilon \mathbf{R}^{n \times n}$ be invertible matrix. Use elimination technique for computation of complexity in the LU decomposition of A. **4**

(c) Describe the singular value decomposition for $\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ **5**

2. (a) State and prove Eckart-Young theorem. **7**

(b) Explain the concept of least squares in four ways. **7**

3. (a) Let $A \epsilon \mathbf{R}^{n \times n}$ be invertible matrix. Define the derivative of A^{-1} and also derive the expression $\frac{dA^{-1}}{dt} = - A^{-1} \frac{dA}{dt} . A^{-1}$ **5+5+4**

(b) Let σ be the singular value of $A \epsilon \mathbf{R}^{m \times n}$. Derive the expression for $\frac{d\sigma}{dt}$.

(c) Derive the expression for the largest Eigen value of $S+T$, where S and T symmetric matrices.

4. (a) Describe the steps involving fast fourier transform. **7+7**

(b) Define circulant matrix. Use fourier transform to obtain its eigen values and eigen vectors.

5. (a) Define Kronecker product for two matrices. Use this definition to prove the following : **5+5+4**

(i) $I_2 \otimes I_3 = I_6$

(ii) $(A \otimes B) \cdot (C \otimes D) = AC \otimes BD$

(iii) $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$

(b) Use the concept of Kronecker sum to derive the eigen value of the Laplacian.

(c) Define vectorization of a matrix and hence prove the following :
Vec $(ABC) = (C^T \otimes A) \cdot \text{Vec}(B)$, where A, B and C are compatible matrices.

6. (a) Factor : $\begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 1 & 2 & 1 \\ 1 & 3 & 6 & 3 \end{bmatrix} = YBZ$, where $Y \epsilon \mathbf{R}^{3\times 2}$, $Z \epsilon \mathbf{R}^{2\times 2}$, $Z \epsilon \mathbf{R}^{2\times 4}$. **5+5+4**

(b) Obtain QR factorization for $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(c) Define Rayleigh coefficient and use it to find the eigen values for
$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

7. (a) Derive the Secular equation with usual notations.

(b) Compute and Interpret the fourier coefficients for $e = (1, 0, 0, 0)^T$. **5+5+4**

(i) Find eigen values and eigen vectors of permutation matrix
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$.

8. (a) Decompose $\begin{bmatrix} 16 & 4 & 2 \\ 0 & 8 & 6 \\ 4 & 5 & 6 \end{bmatrix}$ into sum of rank -1 matrices. **5+5+4**

(b) For $b = (1, 0, 0)$ and $n = 3$. Show that b, Ab, A^2b are non-orthonormal basis for $\mathbf{R}^3$. Use Arnoldi iteration with A to produce an orthonormal basis for $\mathbf{R}^3$.

(c) Obtain the best rank -1 approximation to $\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$.

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