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21MAT4C14L

Sl. No.

M.Sc. IV Semester Degree Examination, October - 2023

MATHEMATICS

Mathematical Methods

(NEP)

Time : 3 Hours Maximum Marks : 70

Note : *Answer* ***any five*** *questions with question No.* ***1 compulsory****.*

1. (a) Solve the following IBVP using the Laplace transform technique. **8+6**

PDE : $u_t = u_{xx}$ $\quad 0 < x < 1,\ t > 0$

BC_S : $u(0, t) = 1,$ $\quad u(1, t) = 1,\ t > 0$

IC : $u(x, 0) = 1 + \sin \pi x,\ 0 < x < 1$

(b) Find the inverse z - transform of :

(i) $\dfrac{z^3 - 20z}{(z-2)^3 (z-4)}$

(ii) $\dfrac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

(iii) $\dfrac{z^2}{(z-1)(z-3)}$

2. (a) Show that $\int_a^x f(t)dt^n = \int_a^x \dfrac{(x-t)^{n-1}}{(n-1)!} f(t)dt$ **7+7**

(b) Obtain the Neumann series solution of the integral equation

$$\phi(x) = 1 + \int_0^x x\, t\, \phi(t)\, dt$$

3. (a) Find the eigen values and eigen functions of the homogeneous integral equation $u(x) = \lambda \int_0^{\pi} k(x, t) u(t) dt$ where $k(x, t) = \begin{cases} \cos x \sin t, & 0 \le x \le t \\ \cos t \sin x, & t \le x \le \pi \end{cases}$ **7+7**

(b) Using Hilbert-Schmidt theorem, solve the following integral equation

$$u(x) = 1 + \lambda \int_0^{\pi} \cos(x + t) u(t) dt$$

P.T.O.

4. (a) Show that Euler's equation (necessary condition for the functional $\int_{x_1}^{x_2} f(x, y, y')$ to be extremum) can be put in the following forms. **8+6**

(i) $\frac{\partial f}{\partial y} - \frac{\partial^2 f}{\partial x \partial y'} - \frac{\partial^2 f}{\partial y \partial y'} y' - \frac{\partial^2 f}{\partial y'^2} y'' = 0$

(ii) $\frac{d}{dx}\left(f - y' \frac{\partial f}{\partial y'}\right) = \frac{\partial f}{\partial x}$

(iii) $f - y' \frac{\partial f}{\partial y'} = C$ (C being a constant), when f does not contain x explicitly

(iv) $y'' \frac{\partial^2 f}{\partial y'^2} = 0$ when f is independent of both x and y.

(b) Prove that catinary is the curve which when rotated about a line generates a minimal surface of revolution.

5. (a) Solve the following Vander pol oscillatory equation $y'' + \varepsilon(y^2 - 1)y' + y = 0$ with $y(0) = A$, and $y'(0) = 0$ using perturbation method. **7+7**

(b) Find the 2-term valid solution of the equation $\varepsilon y'' + (1 + x)y' + y = 0$ with $y(0) = 1$ and $y(1) = 1$.

6. (a) Form an integral equation corresponding to the differential equation given by $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with initial conditions $y(0) = 1,\ y'(0) = 0$. **7+7**

(b) Solve the following integral equation $u(x) = x + \lambda\int_0^{\pi}(1 + \sin x \sin t)u(t)dt$ by the method of separable kernel.

7. (a) Establish Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ in its usual form. **7+7**

(b) Discuss the steps involved in solving a differential equation using perturbation method using an example.

8. (a) Explain the method of finding the solution of homogeneous Fredholm integral equation of the second kind with separable kernel. **5+5+4**

(b) Show that the function $u(x) = xe^x$ is a solution of the volterra integral equation

$$u(x) = \sin x + 2\int_0^x \cos(x - t)u(t)dt$$

(c) Find the extremal of the integral $\int_{x_1}^{x_2}(x^2y'^2+6y^2+2xy)dx$

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