

**M.Sc. IV Semester Degree Examination, October - 2023****MATHEMATICS****Advanced Fluid Mechanics****(NEP)**

Time : 3 Hours

Maximum Marks : 70

**Note :** Answer **any five** questions with question No. **1** is **compulsory**.

1. (a) Explain Stoke's law of viscosity and hence derive the relation between stress **8+6** and rate of strain components.

(b) The state of stress at a point is given by the stress tensor  $\sigma_{ij} = \begin{bmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{bmatrix}$ .

Where a, b, c are constants and  $\sigma$  is some stress value. Determine the constants a, b and c. So that the stress vector on the octahedral plane.

$$\mathbf{n} = \left( \frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k \right) \text{ vanishes.}$$

2. Define the law of conservation of momentum and derive the Navier - Stroke **14**

equation in the form  $\rho \left[ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{q}$ .

3. (a) State and prove Buckingham- $\pi$  theorem. **7+7**

- (b) Define the similarity of two flows and also explain the physical significance of the following non-dimensional parameter.

(i) Reynold's number

(ii) Mach number

(iii) Prandtl number

4. Define the plane Poiseuille flow. For such a flow obtain the velocity distribution, **14** average velocity, maximum velocity, mass flow rate and skin friction.



5. (a) Define : 7+7
- (i) Displacement thickness
- (ii) Momentum thickness
- (iii) Energy thickness,
- Show that :
- (i)  $\int_0^{\delta} \left( \frac{u}{v} \right) dy = \delta - \delta_1$
- (ii)  $\int_0^{\delta} \left( \frac{u}{v} \right)^2 dy = \delta - \delta_1 - \delta_2$
- (iii)  $\int_0^{\delta} \left( \frac{u}{v} \right)^3 dy = \delta - \delta_1 - \delta_3$
- (b) Discuss the Blasius equation for boundary layer on a flat plate and calculate the coefficient of skin friction.
6. (a) Define Circulation. Show that  $\frac{d\Gamma}{dt} = V\nabla^2 T$ . 7+7
- (b) Define Reynold's number and indicate it's significance.
7. (a) Discuss the velocity distribution in the case of the Hagen-Poiseuille flow of a viscous incompressible fluid. 7+7
- (b) Derive the two-dimensional thermal boundary layer equations for flow over a flat plate using the order of magnitude approach.
8. (a) Show that the time rate of change of circulation in a closed circuit, drawn in a viscous incompressible fluid under the action of conservative forces, moving with fluid depends only on the kinematic viscosity and the space rate of change of vorticity components at the contour. 5+5+4
- (b) Find the velocity distribution for plane Couette flow of a viscous incompressible fluid.
- (c) Find Displacement thickness and Momentum thickness for the velocity distribution in the boundary layer given by,  $\frac{u}{v} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ .

