

M.Sc. IV Semester Degree Examination, October - 2023

MATHEMATICS

Applications of Numerical Linear Algebra

(NEP)

Time : 3 Hours

Maximum Marks : 70

Note : Answer *any five* questions with question number **1 compulsory**.

1. (a) Define Kronecker Sum and explain its applications.
(b) Write a note on lowpass filters in signal processing. **7+7**

2. (a) Illustrate Kirchhoff's laws with suitable graph and Laplacian.
(b) Let X be the random variable counting no. of heads in flipping 3 fair coins (iid). Then compute mean and variance of X .
(c) Derive the expression of Poisson distribution with usual notations from Binomial distribution. **5+5+4**

3. (a) Define exponential distribution. Compute its mean and variance. Also illustrate usage of exponential distribution.
(b) Write a note on Markov's and Chebyshev inequalities. **7+7**

4. (a) Let K be a convex set. If f is strictly convex, then prove that there exists at most one local minimum of f in K . Also, prove that if it exists it is the unique global minimum of f in K .
(b) Solve the following problem : **7+7**

$$\min f(x) = 3x_1^2 + 2x_1x_2 + x_1x_3 + 2.5x_2^2 + 2x_2x_3 + 2x_3^2$$

$$\text{Subject to : } x_1 + x_3 = 3$$

$$x_2 + x_3 = 0$$

5. Answer the following.
- (a) Define saddle point. Discuss the nature of saddle points for $f(x, y) = x.y$.
 - (b) Write a note on Newton's method and gradient descent method for optimization problems.
 - (c) Apply the Gradient descent method to compute argmin $x^2 + y^2$ over a rectangle : $[-1, 1] \times [-1, 1]$. **14**
6. (a) Write a note on the following concepts :
- (i) Neural Net
 - (ii) Activation function
 - (iii) Learning function
 - (iv) Bias and variance
- (b) Count the flat pieces in the graph $F(x, y, z) = \text{Relu}(x) + \text{Relu}(y) + \text{Relu}(z)$. **8+6**
7. (a) Write a note on construction of deep neural nets.
- (b) Write a note on weighted least squares. **7+7**
8. (a) Illustrate neural nets to universal approximation through example.
- (b) Describe the applications of Haar wavelet transform.
- (c) Prove-or-disprove : maximum of two convex functions is convex. **5+5+4**

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