No. of Printed Pages : **3**

21MAT4E3CL

Sl. No.

M.Sc. IV Semester Degree Examination, October - 2023

MATHEMATICS

Graph Theory

(NEP)

Time : 3 Hours Maximum Marks : 70

Note : *Answer* ***any five*** *questions with question No.* ***1 compulsory****.*

1. (a) Define factorization of a graph. Find 1-factors of K_6. **5+4+5**

(b) If 'α_0' is the point covering number. 'β_0' is the point independence number of a connected graph 'G' of order 'P' then show that, $\alpha_0 + \beta_0 = P$.

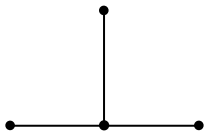
(c) Prove that a point '*v*' is critical in a graph 'G' if and only if some minimum point cover contains *v*.

2. (a) Prove that there are exactly five polyhedra. **7+7**

(b) Define : (i) Linear graph

(ii) Sub-division graph and

(iii) Total graph. Find $L^2(G)$, T(G) and S(G) of the following graph.



3. (a) If 'G' is a graph on P-vertices, then prove that : **7+7**

(i) $2\sqrt{P} \le X(G) + X(\overline{G}) \le P+1$

(ii) $P \le X(G) \cdot X(\overline{G}) \le \frac{(P+1)^2}{4}$

(b) Define chromatic polynomial of a graph. Prove that a graph 'G' is a tree with P-points if and only if $f(G,t) = t(t-1)^{P-1}$

P.T.O.

4. (a) Find the graph 'G' which has adjacency matrix. **5+4+5**

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$ and find the incidence matrix.

(b) Find the eigen values of $K_{1,2}$.

(c) If 'A' is the adjacency matrix of a graph 'G' with p-vertices and q-edges and $\lambda_1, \lambda_2 \ldots . \lambda_P$ are the eigen values of 'G', then prove that $\sum_{P=1}^{P} \lambda_i = 0$ and $\sum_{i=1}^{P} \lambda_i^2 = 2q$.

5. (a) Define automorphism graph. Prove that the at of all automorphisms of a simple graph 'G' is a group with respect to the composition of mappings as the group operations. **5+4+5**

(b) Prove that for any simple graph 'G' $\Gamma(G) = T(\overline{G})$.

(c) Define Line-group of 'G' and explain it by an example.

6. (a) If 'G' is (p, q) graph whose points have degree 'di' then show that the Line graph L(G) has q-points and q_L lines, where $q_L = -q + \frac{1}{2}\sum_{i=1}^{P} d_i^2$ **5+4+5**

(b) Define : (i) Colouring of a graph;
(ii) Chromatic number.

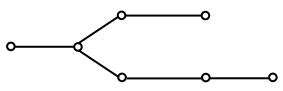
Find chromatic number of K_P, $\overline{K}_P$, $K_{m,n}$ and C_D.

(c) State and prove Euler's formula for connected plane graph.

7. (a) For any (p, q) graph 'G' with incidence matrix B. Prove that $A(L(G)) = B^T B - 2Iq$ where L(G) is the Line graph and 'A' is adjacency matrix. **5+5+4**

(b) Find the spectrum and energy of the graph $K_2 \cup K_3$.

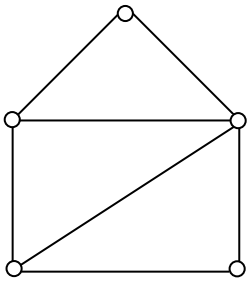
(c) Define identity graph. Show that the following graph is an identity graph.



8. (a) If 'G' is a (p, q) plane graph in which every face is n-cycles, then show that **5+5+4**

$$q = \frac{n(P-1)}{n-2}$$

(b) Find chromatic polynomial of following graph and hence find its chromatic number.



(c) Prove that the number of Spanning trees of labelled graph K_5 is 5^{5-2} (i.e. $5^3 = 125$)

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