21MAT4EACL

Sl. No.

M.Sc. IV Semester Degree Examination, October - 2023
MATHEMATICS
Operations Research
(NEP)

Time : 3 Hours   Maximum Marks : 70

Note : Answer **any five** questions with question No.**1** compulsory.

1. (a) Define convex set. Prove that the intersection of any finite number of convex sets is a convex set.   **4+5+5**

   (b) By graphical method solve the following LPP.

   Maximize $Z = 6x_1 + 4x_2$

   Subject to $-2x_1 + x_2 \leq 2$

   $x_1 - x_2 \leq 2$

   $3x_1 + 2x_2 \leq 9$

   $x_1, x_2 \geq 0$

   (c) Use simplex method to solve the following LPP.

   Minimize $Z = x_2 - 3x_3 + 2x_5$

   Subject to $3x_2 - x_3 + 2x_5 \leq 7$

   $-2x_2 + 4x_3 \leq 12$

   $-4x_2 + 3x_3 + 8x_5 \leq 10$

   $x_2, x_3, x_5 \geq 0$

2. (a) Prove that a necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that total supply equals total demand.   **4+5+5**

   (b) Obtain the initial basic feasible solution of a transportation problem using North West corner rule whose cost and rim requirement table is given below.

Origin/ Destination	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

(c) Find the initial basic feasible solution to the following transportation problem using Row - Minima method.

Source/ Destination	D_1	D_2	D_3	D_4	Supply
S_1	6	3	5	4	22
S_2	5	9	2	7	15
S_3	5	7	8	6	8
Demand	7	12	17	9	

3. (a) What is an assignment problem ? Explain the general Mathematical **6+8** formulation of assignment problem.

(b) A machine operator processes five types of items on his machine each week and must choose a sequence for them. The set-up cost per change depends on the items presently on the machine and the set-up to be made, according to the following matrix.

To Item

$$
\begin{array}{c c} & \begin{array}{c c c c c} A & B & C & D & E \end{array} \\ \text{From Item} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} \infty & 4 & 7 & 3 & 4 \\ 4 & \infty & 6 & 3 & 4 \\ 7 & 6 & \infty & 7 & 5 \\ 3 & 3 & 7 & \infty & 7 \\ 4 & 4 & 5 & 7 & \infty \end{bmatrix} \end{array}
$$

If he processes each type of item only once in each week, how should he sequence the total set-up cost ?

4. (a) Explain Floyd's algorithm for determining the shortest route in the network. **7+7**

(b) Use Dijkstra's algorithm to determine a shortest path from A to C for the following network.
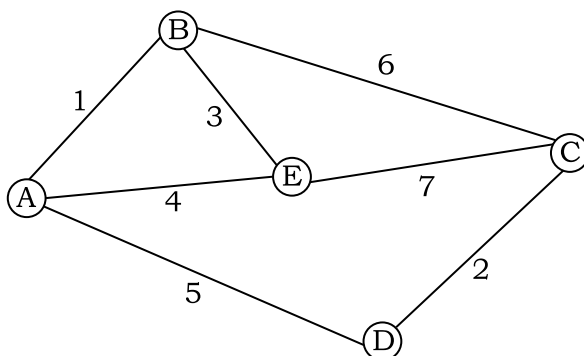
5. (a) Using Gomory's cutting - plane method, find the optimum integer solution to **7+7**
the following LPP.

Maximize $Z = x_1 + x_2$

Subject to the constraints

$3x_1 + 2x_2 \leq 5$

$x_2 \leq 2$

$x_1, x_2 \geq 0$ and are integers.

(b) Explain Mixed integer cutting plane procedure.

6. (a) Find the initial solution to the following transportation problem using Vogel's **7+7**
Approximation method.

Factory/Destination	D_1	D_2	D_3	D_4	Supply
F_1	3	3	4	1	100
F_2	4	2	4	2	125
F_3	1	5	3	2	75
Demand	120	80	75	25	

(b) In the modification of a plant layout of a factory, four new machines M_1, M_2, M_3 and M_4 to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of machine i at place j (in hundred Rupees) is shown below.

Location

	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	–	10	9
M_3	–	11	14	11	7
M_4	14	8	12	7	8

(Machine)

Find the optimal assignment schedule.

7. (a) Determine the maximal flow in the network shown below. **7+7**
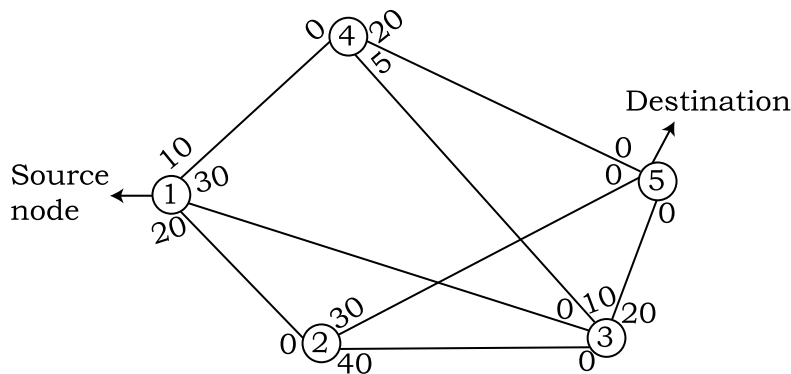
(b) Solve the following LPP.

Maximize $\quad Z = 4x_1 + 6x_2 + 2x_3$

Subject to $\quad 4x_1 - 4x_2 \leq 5$

$\quad\quad\quad\quad -x_1 + 6x_2 \leq 5$

$\quad\quad\quad\quad -x_1 + x_2 + x_3 \leq 5$

$\quad\quad x_1, x_2, x_3 \geq 0$ and

$\quad\quad x_1, x_3$ are integers.

8. (a) Explain the Branch - and - Bound algorithm for solving mixed integer programming problem. **5+5+4**

(b) There are four jobs to be assigned to five machines only one job can be assigned to one machine. The amount of time in hours required for the jobs per machine are given in the following table.

Jobs	Machines				
	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

Find an optimum assignment of jobs to the machines to minimize the total processing time and also find out which machine no job is assigned. What is the total processing time to complete all the jobs.

(c) Determine an initial basic feasible solution for the following Transportation problem using least cost method.

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	

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