B.Sc. I Semester (NEP) Degree Examination, March/April - 2022

MATHEMATICS

Paper No. 1 DSC - 1 - Fundamentals of Algebra and Calculus

Time : 3 Hours   Maximum Marks : 60

Instruction : *(i)*   *Answer **all** questions from **Section-A**.*

      (ii) *Answer **any four** questions from **Section-B**.*

      (iii) *Answer **any two** full questions from **Section-C**.*

SECTION - A

1. Answer the following sub-questions, each sub-questions carries **one** mark. **10x1=10**

   (a) Define Eigen value and Eigen vector of a square matrix.

   (b) Find the Rank of the Square Matrix A.

   Where, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.

   (c) Find the Angle between radius vector and the tangent for $r^2 = a^2.\cos 2\theta$.

   (d) Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \dfrac{\pi}{2}$.

   (e) Find the left hand and right hand limits of $f(x) = |x|$ As x tends to 0.

   (f) State Rolle's Theorem.

   (g) Find the nth derivative of $\sin^3 x$.

   (h) If $y = \log(x^2 - 4)$, then find y_n.

   (i) State Cayley - Hamilton Theorem.

   (j) Evaluate : $\displaystyle \lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^2.\sin x}$.

SECTION - B

Answer **any four** of the following questions, each question carries **five** marks.

4x5=20

2. Using Cayley - Hamilton's Theorem find A^{-1} if, $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

3. Show that the pair of Circles Intersect Orthogonally :
$r^n = a^n.\cos n\theta$, $r^n = b^n.\sin n\theta$.

4. Verify Lagrange's Mean Value Theorem for $f(x) = x^2 - 3x - 2$ in $[-2, 3]$.

5. If $y = \sin(m.\sin^{-1}x)$ then prove that, $(1-x^2)y_{n+2} - (2n+1)\, x\, y_{n+1} + (m^2 - n^2)\, y_n = 0$.

6. Find the evolute of the parabola $y^2 = 4ax$.

7. Find the real values of λ, for which the system,
$x + 2y + 3z = \lambda x$
$3x + y + 2z = \lambda y$
$2x + 3y + z = \lambda z$.
have non-zero solutions.

SECTION - C

Answer **any three** of the following questions, each question carries **ten** marks.

3x10=30

8. (a) Reduce the Matrix A to its normal form where, **5**

$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$.

And hence find the Rank of Matrix.

(b) Verify the following system of equations is consistent. Solve if consistent. **5**
$x + 2y - z = 1$
$3x + 8y + 2z = 28$
$4x + 9y - z = 14$

9. (a) Find the angle of intersection of the pair of curves. **5**
     $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.

   (b) Derive derivative of Arcs in cartisian form and parametric form. **5**

10. (a) State and prove Cauchy's Mean Value Theorem (Second Mean Value **6**
     Theorem).

   (b) Obtain expansion of e^x as an infinite series (Mac Laurin's) **4**

11. (a) State and prove Leibnitz Theorem. **4**

   (b) Trace the curve $y^2(a - x) = x^3$, $a > 0$. **6**

12. (a) Show that the pair of curves intersect orthogonally. **6**
     $r = a(1 + \sin\theta)$, $r = b(1 - \sin\theta)$

   (b) Find the Pedal equation $(p - r)$ equation of the curve. **4**
     $r = a(1 - \cos\theta)$

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