CWM-1.3

Ph.D Course Work Examinations, July-2023

MATHEMATICS

Course-III -1.3: Mathematics

Time: 3 Hours **Max.Marks:70**

Instructions to Candidates: (i) Answer any Five full questions.

(ii) Each question carries equal marks.

1(a) State and prove Bolzano-Weierstrass theorem for sequences.

(b) State and prove the following inequalities: (6+8)

i. Young's inequality

ii. Bessel's inequality

2(a) Prove or disprove: If $f_n : [a,b] \to R$ is a sequence of integrable functions and converges pointwise to f then f is integrable over $[a,b]$.

(b) If $C[0,1]$. is the set of all real valued continuous function on $[0,1]$., show that $C[0,1]$. is a complete metric space under the metric: (7+7)

$$d(f,g) = \underset{x\in[0,1]}{Max}|f(x) - g(x)|$$

3(a) State and prove Fundamental theorem of Homomorphism.

(b) Prove that every square matrix satisfies its characteristic equation.

(c) Let V be the vector space of all 2×2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V which has 4 elements. (5+5+4)

4(a) With the help of Cayley-Hamilton the theorem find A^6, Where

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$$

(b) In the vector space R^3 express the vector $(1,\ -2,\ 5)$ as a linear combination of the vector $(1,1,1),(1,2,3)$ and $(2,-1,1)$.

(c) Consider $T:\ P_2(R) \to P_4(R)$ given by $T[P(x)] = P'(x) + \int_0^x P(t)\,dt$ w.r.t $\{1,x,x^2,x^3\}$ and $\{1,x,x^2,x^3,x^4\}$ as standard basis of $P_3(R)$ and $P_4(R)$ find transformation matrix. (5+4+5)

5(a) Solve the following using Lagrange method

(i) $y^2(x-y)p + x^2(y-x)q = z(x^2+y^2)$.

(ii) $yzp + 2xq = xy$.

(b) Solve the following using Charpit's method (7+7)

(i) $p^2 + q^2 = z^2(x+y)$.

(ii) $p^2+q^2=z^2(x+y)^2$.

6(a) Find the equation of the system of surface which cut orthogonally the cones of the system $x^2+y^2+z^2=cxy$

(b) Show that the equations $xp-yq=0$ and $z(zp+yq)=2xy$ are compatible and solve them. (7+7)

7(a) Using Euler's and Modified Euler's method compute $y(2)$ from.

$$\frac{dy}{dx}=\frac{y-x}{y+x}; \quad y(0)=1.$$

(b) Evaluate the approximate value of $\int_0^{\pi/2} \mathrm{Sin}(x)\,dx$ by (a) Trapezoidal rule (b) Simpson's 1/3rd rule (c) Simpson's 3/8th rule. (7+7)

8(a) Estimate the value of $y(1.05)$ using the Hermite interpolation formula from the following data.

x	y	y'
1.00	1.00000	0.50000
1.10	1.04881	0.47673

(b) Discuss the rate of convergence of the Newton-Raphson method. (7+7)

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