CWM-1.4 (D)

Ph.D. Course Work Examinations, July-2023

MATHEMATICS

Course-IV-1.4 (D): Wavelets

Time: 3 Hrs **Max.Marks:70**

Instructions: Answer any five questions and each question carries equal marks.

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1. (a) Consider the set $V = \{a \ . \ t / t \in (0,1)\}$. Which of the following is a vector space? **[5+5+4]**

(i) $a \in (0,1)$ (ii) $a \in £$ (iii) $a \in ¤$

(b) Find the distances between two vectors s and g with respect to metrics: d_∞, d_1, d_2 *and* d_3.
Where $s(n) = \delta(n) + 2\delta(n-1) + 4\delta(n-2) + \delta(n-3)$,
$g(n) = \delta(n) + 2\delta(n-1) - 4\delta(n-2) - \delta(n-3)$ with
$\delta(n)$: *Dirac delta sequence / function.*

© Find a basis for the subspace spanned by the columns in the matrix: $\begin{pmatrix} -2 & 4 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & 1 & 8 \\ 2 & 4 & 15 \\ 3 & 9 & 15 \end{pmatrix}$

2. (a) Use the standard inner product in ¡ ^{3} to obtain the reciprocal basis corresponding to a

basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(b) Let P_n be the set of all polynomials of degree less or equal to n over ¡ . Show that

$\gamma = \{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$ is a basis of P_3. Also find the coordinates of

$5 - x + 2x^2 + 8x^3$ with respect to γ. **[7+7]**

3. (a) Suppose $f : £^{2\times 2} \to £^{2\times 2}$ is a map defined by $f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a-b & -b \\ -c & d-c \end{bmatrix}$. Show that f is

linear and also find its matrix representation with respect to standard basis.

(a) Define LTI system and illustrate its applications. **[7+7]**

4.(a) Illustrative the fast Fourier transform through matrix decomposition view.

(b) Use eight-point fast Fourier transform as down sampling operation to compute the FFT of $\{1,2,3,4,5,6,7,8\}$. **[7+7]**

5. (a) Let $s(t) = -u(t) + 3u(t-0.5) - 2u(t-1)$, where $u(t)$ is the unit step function. Express $s(t)$ as linear combination of $\{1,1\}$ and $\{1,-1\}$.

(b) Construct and plot the wavelet basis functions for the space V_3. Where V_3 is the set of all Piecewise constant functions over $[0,1] = \left[0,\frac{1}{8}\right] \cup \left[\frac{2}{8},\frac{3}{8}\right] \cup \ldots \cup \left[\frac{7}{8},1\right]$. **[7+7]**

6. (a) Determine the minimum sampling rates for the following signals:

(i) An audio signal with bandwidth 9kHz

(ii) A sinusoid $20\sin(80\pi t)$

(iii) A mixture of signal and noise, where the signal is bandlimited to 20kHz, and the noise is white.

(b) Find the Haar Wavelet transform of $f(t) = \begin{cases} \frac{1}{4}, & t \in [0, 0.25) \\ \frac{-1}{4}, & t \in [0.25, 0.5) \\ \frac{-1}{\sqrt{2}}, & t \in [0.5, 0.75) \\ \frac{3}{2\sqrt{2}}, & t \in [0.75, 1) \end{cases}$. **[7+7]**

(7) (a) Write a detail note on Haar Multiresolution Analysis

(b) Up sample $\{1,2,1,0,-1,1\}$ by 3 and convolve with $\{1,1,1,-1\}$ **[10+4]**

(8) Write a detail notes on Quadrature Mirror Filters.

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