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Sl. No.

M.Sc. I Semester Degree Examination, April/May - 2024

MATHEMATICS

DSC - 1 : Algebra

(NEP)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　Maximum Marks : 70

Note : *Answer **any five** of the following questions with question No. **1** is **Compulsory**. Each question carries **equal** marks.*

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1. (a) Let H be a normal subgroup of group G and $\phi : G \to G/H$ is a mapping by $\phi(a) = Ha \; \forall \; G$. Then show that ϕ is epimorphism with $\text{Ker}\phi = H$. **4**

(b) Show that centre of group G is characteristic subgroup of G. **4**

(C) State and prove Born-Side Lemma. **6**

2. (a) State and prove Sylow's 2^{nd} Theorem. **5**

(b) Prove that every abelian group is always solvable. **4**

(c) Show that no group of order 30 is simple. **5**

3. (a) Prove that A non-zero commutative ring with unity is a field if it has no proper ideal. **4**

(b) Let $R[x]$ be the polynomial ring over R then prove that : **6**

(i) R has identity if and only if $R[x]$ has identity.

(ii) If R is an integral domain if and only if $R[x]$ is an integral domain.

(c) Show that any field F is an Euclidean Domain. Discuss the converse. **4**

4. (a) Find the gcd of $x^4 + x^3 + 2x^2 + x + 1$, $x^3 - 1$. **3**

(b) If $f(x) \in z[x]$ is primitive and $f(x)$ is irreducible over z, then prove that F is irreducible over Q. **6**

(c) Show that $R[x]$ is a unique Factorization Domain then R be an integral Domain with unity is a unique Factorization Domain. **5**

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5. (a) Show that if $F \subseteq K$, then $\alpha \in K$ is algebraic over F if and only if $F(\alpha)$ is a finite extension of F. **5**

(b) Show that Prime subfield of a field is isomorphic to either Q or Z_p for some prime. **5**

(c) Find $\left[Q\left(\sqrt{2} + \sqrt{3}\right); Q\right]$ and find a basis for **4**

$$Q\left(\sqrt{2} + \sqrt{3}; Q\right) = \left[Q\left(\sqrt{2} ; \sqrt{3}\right); Q\right].$$

6. (a) Show that Q_8 is a solvable group. **4**

(b) Prove that every finite group having atleast two elements has a composite series. **5**

(c) Prove that characteristic of an integral domain R is either zero or prime. **5**

7. (a) Verify the primitiveness of $f(x) = 4x^3 + 6x + 2$ in $z_n(x)$. **4**

(b) If L is a finite extension of K and K is a finite extension of F then prove that L is a finite extension of F. **6**

(c) Show that $x^4 + 3x^3 - 9x^2 + 18x - 12$ is irreducible over Q. **4**

8. (a) Show that A_4 has no subgroups of order 6. **5**

(b) State and prove the Einstein criterion of irreducibility over UFD. **5**

(c) Show the splitting field of $x^4 + 1$ over $\left(\sqrt{2}, i\right)$ whose degree over Q is 4. **4**

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