No. of Printed Pages : **2**

21MAT1C3L

Sl. No.

M.Sc. I Semester Degree Examination, April/May - 2024

MATHEMATICS

Differential Equations

(NEP)

Time : 3 Hours

Maximum Marks : 70

Note : *Answer* ***any five*** *questions with question No.* ***1 (Q.1) Compulsory****.* ***All*** *questions carries* ***equal*** *marks.*

1. (a) Prove that n solutions $y_1, y_2, \ldots\ldots y_n$ of $L_n(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots\ldots + a_n\, y = 0$ where $a_1, a_2, \ldots\ldots a_n$ are constants are linearly independent in the interval I. If and only if $W\,(y_1, y_2, \ldots\ldots y_n)\,(x) \neq 0, \forall x \in I$. **5**

(b) Solve the initial value problem $y'' + 2y' - 3y = 0,\ y(0) = 0$ and $y'(0) = 1$ **4**

(c) Discuss the existence and uniqueness theorem for second order homogeneous linear differential equation with constant coefficients. **5**

2. (a) If $x(t)$ be any solution of $x'' + a(t)x = 0$ defined on $(0, \infty)$ and $a(t)$ be negative then prove that the solution $x(t)$ has at most one zero. **5**

(b) State and prove Sturm's Comparison theorem. **5**

(c) Reduce $x^2y'' - 2(x^2 + x)y' + (x^2 + 2x + 2)y = 0$ to its canonical form and solve. **4**

3. (a) Find all the solutions of boundary value problem $x'' + \lambda x = 0,\ x(0) = 0,\ x(l) = 0$ **6**

(b) State and prove Green's theorem. **8**

4. (a) Find the radius of convergence and exact interval of convergence of the power series $\sum \dfrac{(n+1)\,x^n}{(n+2)(n+3)}$ **4**

(b) Find the power series solution in powers of $(x-1)$ for the IVP $xy'' + y' + 2y = 0,\ y(1) = 2$ and $y'(1) = 4$. **6**

(c) Define ordinary and regular singular point of a differential equation. Show that $x = 0$ is an ordinary point and $x = 1$ is a regular singular point of $(x^2 - 1)y'' + xy' - y = 0$ **4**

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5. (a) Find the critical points. and further determine its nature and stability of the following system. **6**

(i) $\frac{dx}{dt} = 3x + y, \frac{dy}{dt} = x + 3y$

(ii) $\frac{dx}{dt} = 2x + 7, \frac{dy}{dt} = 3x + 8y$

(b) Write notes on : **8**

(i) Centre point (ii) Saddle point

(iii) Spiral point (iv) Node

6. (a) Verify Liouville's theorem for $x^2y'' - xy' + y = 0$. **5**

(b) Verify $x^2y'' - 2xy' + 2ny = 0$ is a self adjoint differential equation. If not transform it into an equivalent self adjoint form. **5**

(c) Prove that two linearly independent solutions $x_1(t)$ and $x_2(t)$ of $x'' + a(t)x + b(t)x = 0$ have no common zero's. **4**

7. (a) Construct the Green's function for the boundary value problem **5**

$x'' + K^2x = 0,\ x(0) = 0$ and $x(l) = 0$

(b) Show that the function $f_1(x) = 4$ and $f_2(x) = x^3$ are orthogonal over $(-2, 2)$ and determine the constant A and B such that the function $f_3(x) = 1 + Ax + Bx^2$ is orthogonal to both $f_1(x)$ and $f_2(x)$ **5**

(c) Define the following : **4**

(i) Power series

(ii) Radius of convergence

(iii) Orthogonal function

(iv) Orthogonal set of functions

8. (a) Solve $y'' - 2y' - 12y = 2x^3 - x + 3$ by the method of undetermined Coefficients. **5**

(b) Discuss the method to find the power series solution about $x = 0$ is an ordinary point of $y'' + P(x)y' + Q(x)y = 0$. **5**

(c) Describe Liapunov's direct method to discuss the stability of the system. **4**

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