

M.Sc. I Semester Degree Examination, April/May - 2024**PHYSICS****Mathematical Methods of Physics****(NEP)**

Time : 3 Hours

Maximum Marks : 70

Note : Answer **any five** of the following questions with Question No. **1** (Q1) is **compulsory**, each question carries **equal** marks.

1. (a) Solve the Legendre differential equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, by power series method; where n is a constant. **10**
- (b) Prove the following Hermite recurrence relation **4**

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$
2. (a) Find the Fourier series of the following function : **5**

$$f(x) = \begin{cases} 0 & \text{when } -\pi \leq x \leq 0 \\ k & \text{when } 0 \leq x \leq \pi \end{cases}$$
- (b) What are cosine and sine series ? Give an example of an even function and find its cosine series. **3**
- (c) Find the inverse Fourier transform of $F(s) = \frac{s^2}{(s^2+a^2)^2}$ **6**
3. (a) What is a Hilbert space ? Explain its properties, including completeness and the existence of an orthonormal basis. **5**
- (b) Define complex vector space and show that $S = [(i, 0, 0), (0, i, i), (0, 0, 1)]$ is a function of C^3 . **5**
- (c) What are Unitary and Hermitian matrices ? Give an example of each. **4**
4. (a) Define an analytic function. Show that $f(z) = |z|$ is not analytic. **5**
- (b) State and prove Cauchy's integral theorem for a simply connected region. **6**
- (c) What is vector analysis ? Explain the difference between Cartesian and Curvilinear coordinate systems. **3**

5. (a) Explain the reducible and irreducible representations of a group with an example. **7**
- (b) What are Christoffel Symbols ? Obtain the Christoffel's Symbols of I and II kind. **7**
6. (a) What are Fourier transforms ? Give the properties of Fourier transforms. **2**
- (b) What is the convolution theorem for Laplace transforms ? Explain how it is used to solve integral equations. **6**
- (c) Explain the Algebra of linear operators. **6**
7. (a) What are the functions of the complex variable ? Explain. **4**
- (b) State and prove Gauss divergence theorem. **6**
- (c) Discuss about covariant and contravariant tensors with an example. **4**
8. (a) Obtain the Laplace transform of the function $f(t) = \sinh at \sin at$. **5**
- (b) Find the work done in moving a particle in the force field $\vec{F} = 3x^2 \hat{i} + (2xz - y) \hat{j} + z \hat{k}$ along the curve defined by $x^2 = 4y$ and $3x^2 = 8z$ from $x=0$ to $x=2$. **5**
- (c) What is the three-dimensional rotation group $SO(3)$ and show how it is related to the group $SU(2)$? **4**

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