21MAT3C11L

M.Sc. III Semester Degree Examination, April/May - 2024
MATHEMATICS
Mathematical Modelling
(NEP)

Time : 3 Hours   Maximum Marks : 70

Note : *Answer **any five** questions with Question Number **Q.1 compulsory**.*

1. (a) Explain the simple compartment model interms of linear differential equations of first order.

(b) In logistic law of population growth is given by $\frac{dx}{dt}=x(a-bx)$, a>0, b>0, if a$=0.03134$, b$=1.5887\times10^{-10}$, $x(0)=39\times10^{6}$, show that

$$x(t)=\frac{313,400,000}{1.5887+6.44719\times e^{-0.03134t}}$$

If time zero corresponds to 1790. estimate the population in 1800, 1850, 1900. Find the year of point of inflexion and the limiting population.   **7+7**

2. (a) Explain mathematical modelling of Multi-species model through system of ordinary differential equations of first order.

(b) Develop a model for diabetes mellitus in terms of system of ordinary differential equations.   **7+7**

3. (a) Show that if the gravitational force (central force) experienced by a particle moving around the center of force obeys inverse square law then the path of the particle is a conic section with the center of force at one focus.

(b) Deduce Kepler's three laws of planetary motion, when the law of attraction is the inverse square law.   **8+6**

4. (a) Describe the methodology for obtaining complementary function using matrices.

(b) Solve the following difference equations
(i) $x_{t+2}-7x_{t+1}+12x_{t}=3^{t}+t^{4}+4^{t}t^{3}$.
(ii) $x_{t+2}-4x_{t+1}+3x_{t}=t$ **6+8**

5. (a) Describe the application of directed graphs in detection of cliques.

(b) Explain the importance of directed graphs in finding whether one can introduce one-way traffic on some or all roads of the city without preventing persons going from any point of the city to another point. **7+7**

6. (a) Suppose that the population x(t) and y(t) satisfy the model given by $dx/dt = 1 - xy$, $dy/dt = x - y$. Determine all the critical points of the system and discuss the type and stability of each of these critical points.

(b) Explain the importance of linear differential equations of second order in electrical system involving an electrical circuit with a resistance, inductance, capacitance and battery voltage. **7+7**

7. (a) Describe the cobweb model for demand and supply of product through difference equations.

(b) Write a brief explanation on matrices associated with a directed graph. **7+7**

8. (a) Find the expression for the radial and transverse components of velocity and acceleration of the particle moving around a center of force in elliptic path.

(b) Explain any two mathematical models interms of directed graphs.

(c) Solve the following simultaneous difference equations

$x_{n+1} - x_n + 2y_{n+1} = 0$

$y_{n+1} - y_n - 2x_n = 2^n$ **5+5+4**

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