

M.Sc. III Semester Degree Examination, April/May - 2024

MATHEMATICS

Matrix Computations

(NEP)

Time : 3 Hours

Maximum Marks : 70

Note : Answer **any five** questions with **Q.No. 1 Compulsory**. Each question carry **14** marks.

1. (a) Compute the four fundamental subspaces associated with the matrix **5+5+4=14**
$$A = \begin{bmatrix} 2 & 10 \\ 3 & 15 \end{bmatrix}$$
- (b) (i) Let $A, B \in \mathbf{R}^{m \times n}$. Then prove the following :
Rank of $A+B \leq (\text{rank of } A) + (\text{rank of } B)$
(ii) If A is m by r and B is r by n both with rank r -then AB also has rank r .
- (c) Let $A \in \mathbf{R}^{m \times n}$. Discuss the relationship between the row space and null space of A .
2. (a) Express $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 8 & 7 & 8 \end{bmatrix} = l_1 u_1^T + l_2 u_2^T + l_3 u_3^T$. **6+8=14**
- (b) Describe the singular value decomposition for $\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$
3. (a) State and prove Eckart-Young theorem. **7+7=14**
(b) Prove that $A \in \mathbf{R}^{m \times n}$ has SVD.
4. (a) Write a detailed note on least- squares in four ways involving the data analysis.
(b) Use Arnoldi's iteration to compute Hessenberg matrices and illustrate with one example. **7+7=14**
5. (a) Let $A \in \mathbf{R}^{n \times n}$ be invertible matrix. Define the derivative of A^{-1} and also derive the expression : $\frac{d}{dt}(A^{-1}) = -A^{-1} \frac{d}{dt}(A) A^{-1}$. **5+5+4=14**
(b) Derive the expression for the largest eigenvalue of $S+T$, where S and T are same size of symmetric matrices.
(c) Obtain the relation between discrete Fourier transform matrix and the Fourier matrix.

6. (a) Define the Raylight quotient. Use it to find the dominant singular value and hence singular vector. **5+5+4=14**
- (b) Use the Concept of Kronecker sum to derive the eigenvalue of Laplacian.
- (c) Define vectorization of a matrix and hence prove the following :
 $\text{Vec}(ABC) = (C^T \otimes A) \cdot \text{vec}(B)$, where A, B and C are Compatible matrices.
7. (a) Compute and interpret the Fourier coefficients for $(0, 1, 0, 0)^T$. **5+5+4=14**
- (b) Derive the expression for a relation between singular values and eigen values of a square matrix.
- (c) Write a note on Householder reflections.
8. (a) Let S be a real symmetric matrix. Then prove the following are equivalent :
 (i) S is positive definite **5+5+4=14**
 (ii) All the eigen values of S are positive
- (b) Define Kronecker product of matrices. Use this definition to prove the following :
 (i) $I_2 \otimes I_3 = I_6$
 (ii) $(A \otimes B) \cdot (C \otimes D) = AC \otimes BD$
- (c) Obtain the QR factorization for $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

- o 0 o -

