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21MAT3C12L

Sl. No.

M.Sc. III Semester Degree Examination, April/May - 2024

MATHEMATICS

DSC 12 : Partial Differential Equations

(NEP)

Time : 3 Hours | Maximum Marks : 70

Note : *Answer* ***any five*** *of the following questions with question No.* ***1 (Q.1)*** *is* ***Compulsory****.* ***All*** *question carries* ***equal*** *marks.*

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1. (a) Derive the steps involved for the solution of Lagrange's Linear Partial differential equation of order one. **7**

(b) Form a PDE by eliminating arbitrary constants/functions for the following : **7**

(i) $x+y+z=f(x^2+y^2+z^2)$

(ii) $(x-\mathrm{h})^2+(y-\mathrm{k})^2+z^2=\lambda^2$

2. (a) Find the family orthogonal to $\phi\ [z(x+y)^2,\ x^2-y^2]=0$ **4**

(b) Show that the equations $x\mathrm{p}=y\mathrm{q}$; $z(x\mathrm{p}+y\mathrm{q})=2xy$ are compatable and solve them. **6**

(c) Solve : $\mathrm{p}+3\mathrm{q}=5z+\tan\ (y-3x)$ **4**

3. (a) Use separation of variable method to solve $\dfrac{\partial^2 z}{\partial x^2}=a^2 z$ given that $\dfrac{\partial z}{\partial y}=0$ **5**

At $x=0$ and $\dfrac{\partial z}{\partial x}=\mathrm{a}\sin y$ at $x=0$

(b) Obtain a solution of Heat equation by the separation of variables. **5**

(c) Solve : $2\mathrm{D}^2-5\mathrm{D}\mathrm{D}'+2(\mathrm{D}')^2=24(2y-x)$ **4**

4. (a) Derive a solution of Laplace equation in cylindrical co-ordinates. **7**

(b) Derive the solution of diffusion equation in spherical co-ordinates. **7**

5. (a) Describe Monge's Method of solution of non-linear PDE $\mathrm{Rr}+\mathrm{Ss}+\mathrm{Tt}=\mathrm{V}$. **4**

(b) Solve $\mathrm{t}-\mathrm{r}\ \sec^4 y=2\mathrm{q}\tan y$ by Monge's method. **10**

6. (a) Solve $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} = 0$ **4**

(b) Reduce $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial z}{\partial x} = 0$ to its canonical form. **5**

(c) Transform PDE into canonical form $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ **5**

7. (a) Use the Laplace transform method to solve Initial boundary value problem described as PDE **7**

$$u_{xx} - \frac{1}{C^2}u_{tt} - \cos wt = 0;\ 0 \le x < \infty$$

Boundary conditions $u(0, t) = 0$ initial conditions

$u_t(x, 0) = u(x, 0) = 0$

(b) Solve free vibration of a semi-infinite strings problem with the aid of Fourier Transform **7**

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2\frac{\partial^2 u}{\partial x^2};\ 0 < x < \infty,\ t > 0.$$

IC's : $u(x\,;\,0) = f(x)$ and $\frac{\partial u}{\partial x}(x, 0) = g(x)$

8. (a) Solve the following IBVP using Laplace transform Technique : **5**

$u_t = u_{xx}$; $0 < x < 1$, $t > 0$

BC's ; $u(0, t) = 1$, $u(1, t) = 1$ $t > 0$

IC's : $u(x, 0) = 1 + \sin\pi x$, $0 < x < 1$

(b) Solve : **5**

$(D^2 + DD' + D' - 1)z = \sin(x + 2y)$

(c) Discuss the eigen function method to solve Partial Differential equation. **4**

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