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21MAT1C2L

Sl. No.

M.Sc. I Semester Degree Examination, April/May - 2024

MATHEMATICS

Real Analysis

(NEP)

Time : 3 Hours | Maximum Marks : 70

Note : *Answer **any five** questions with Q No. **1 compulsory**. Each question carries **14** marks.*

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1. (a) Show that (a, b) is uncountable. Where a(<)b$\in$**R**.

(b) let α and β be infimum and supremum of $A(\neq\phi) \subseteq R$. Let K$\in$**R**, define $KA = \{\lambda x : x \in A\}$. Then prove the following :

$$\text{Sup}(KA) = \begin{cases} K \sup A, & \text{if } K > 0 \\ K \inf A, & \text{if } K < 0 \end{cases}$$

(c) Prove - or - disprove : Between two real numbers there are infinite rationals.

5+5+4

2. (a) Compute the following for the subsets of **R** under Euclidean topology on **R**.

(i) interior of Z (ii) derived set of **N**

(iii) exterior of (2, 3) (iv) closure of {2}

(v) boundary of [2, 3]

(b) Prove that every compact set in **R** is closed. Discuss the converse.

(c) Prove - or - disprove : Q is dense in **R**. **5+5+4**

3. (a) Prove the following :

A number C$\in$**R** is a cluster point of a subset A of **R** if and only if there exists a sequence $\{x_n\}$ in A - {c} converges to C.

(b) let f : [0, 1] $\rightarrow$ **R** be continuous. Then prove that f is bounded. Discuss the conditions on domain of f.

(c) State and prove preservation of intervals theorem. **5+5+4**

4. (a) Prove - or - disprove the following statement :

let $\{f_n\}$ be a sequence of differentiable functions and $f_n \rightarrow f$ pointwise, then $f'_n \rightarrow f'$.

(b) State and prove Cauchy's criteria for uniform convergence of series of functions.

P.T.O.

(c) Test the uniform convergence for the following sequence and series of functions.

(i) $f_n(x) = \frac{1}{x^2 + n^3}, x \in [0, 2]$

(ii) $f_n(x) = x^n,\ x \in [0, 1]$

(iii) $\sum \frac{\sin\left(x^4 + n^2\right)}{n^2 (n+8)}$ **5+5+4**

5. (a) If $\{f_n\}$ is a sequence of continuous functions on [a, b] and if $f_n \to f$ uniformly on [a, b], then prove that f is continuous on [a, b].

(b) If a series $\sum f_n$ uniformly converges to f on [a, b], and each f_n is continuous on [a, b], then prove that f is integrable and $\sum \int_a^x f_n(t)dt = \int_a^x f(t)dt., \forall x \in [a,b]$ **7+7**

6. (a) Discuss the compactness and connectedness for the following subsets of **R** under Euclidean topology on **R**.

(i) sin (**R**) (ii) cos ([$-\pi$, π])
(iii) Z (iv) Q^c
(v) [420, 847]

(b) Define compactness. Prove that continuous image of compact set in **R** is compact. **7+7**

7. (a) State and prove Weierstrass M-test.

(b) Classify discontinuities for the following functions :

(i) $f(x) = \begin{cases} x^2 \cos\left(1/x\right) & x \neq 0 \\ 8.8 & x=0 \end{cases}$

(ii) signum function at $x=0$

(iii) $g(x) = \begin{cases} \sin\left(1/x\right) & x \neq 0 \\ 0 & x=0 \end{cases}$ **5+9**

8. (a) Prove that a non-empty subset of **R** which is bounded below has infimum in **R**.

(b) Prove that uniform continuous function is continuous. Discuss the converse.

(c) Let $\{f_n\}$ be a sequence of functions such that $f_n \to f$ pointwise on [a, b] and $M_n = \sup |f_n(x) - f(x)|$. Then $x \in [a, b]$
Prove that $f_n \to f$ uniformly on [a, b] if and only if $M_n \to 0$ as $n \to \infty$. **5+5+4**

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