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21MAT1C5L

Sl. No.

M.Sc. I Semester Degree Examination, April/May - 2024

MATHEMATICS

Topology

(NEP)

Time : 3 Hours | Maximum Marks : 70

Note : *Answer* ***any five*** *of the following questions with question No.* ***1 (Q.1) Compulsory***.

1. (a) If (X, d) be a metric space then prove the followings :

(i) Every open sphere is an open set.

(ii) Every closed sphere is a closed set.

(b) Prove that in a metric space X, a subset A is closed if and only if A contains all its limit points.

(c) Show that every convergent sequence in a metric space is a Cauchy sequence.

5+5+4

2. (a) Let (X, τ) be a topological space and if A, B are subsets of X then prove the followings :

(i) $A \subset B \Rightarrow A^\circ \subset B^\circ$

(ii) $(A \cap B)^\circ \Rightarrow A^\circ \cap B^\circ$

(iii) $(A^\circ \cup B^\circ) \subset (A \cup B)^\circ$

(iv) $(A^\circ)^\circ \Rightarrow A^\circ$

(b) Define a Dense set in a topology. Let (X, τ) be a topological space and $A \subset X$ then prove that A is dense in X if and only if for every non-empty open set G, $A \cap G \neq \phi$. **8+6**

3. (a) Let X and Y be two topological spaces, prove that a map $f : X \to Y$ is continuous if and only if for every $E \subset X$, $f(\overline{E}) \subset \overline{f(E)}$

(b) Let X and Y be two topological spaces. If $f : X \to Y$ be a one - one and onto map then prove that the following are equivalent :

(i) f is homomorphism

(ii) If $G \subset X$ and $f(G)$ is open in Y if and only if G is open in X

(iii) If $F \subset X$ and $f(F)$ is closed in Y if and only if F is closed in X

(iv) If $E \subset X$ then $f(\overline{E}) \subset \overline{f(E)}$ **6+8**

4. (a) Let (x, τ) be a topological space then show that T_0- is a topological property.

(b) Show that every T_2-space is a T_1-space.

(c) Show that a closed subspace of a normal space is normal. **5+4+5**

5. (a) Prove that every compact subset A of a Hausdorff space is closed.

(b) Let (x, τ) be a topological space then prove that X is compact if and only if every collection of closed subsets of X with Finite intersection property is fixed.

(c) Consider the topology $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$ on the set $X = \{a, b, c, d, e\}$. Show that :

(i) (X, τ) is disconnected.

(ii) $Y = \{b, d, e\}$ is connected subset of X. **5+5+4**

6. (a) Define a sub-base for a topology. If S is the non-empty collection of subsets of X, then show that finite intersection of members of S form a base for τ.

(b) State and prove Lindelof theorem. **6+8**

7. (a) Show that a topological space X is completely regular if and only if for every $x \in X$ and every open set G containing x, there exists a continuous mapping f of X into [0, 1] such that $f(x) = 0$ and $f(y) = 1 \ \forall \ y \in X - G$.

(b) Prove that a topological space X is locally connected if and only if the components of every open subspace of X are open in X. **8+6**

8. (a) Show that the topological space X is normal if and only if for any closed set F and open set G containing F, there exists an open set V such that $F \subset V$ and $\overline{V} \subset G$.

(b) Prove that two open subsets of a topological space are separated if and only if they are disjoint.

(c) Show that every second countable space is first countable. **5+5+4**

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