



**B.Sc. I Semester Degree Examination, April/May - 2024**

**MATHEMATICS**

**DSC-1 : Fundamentals of Algebra and Calculus**

**(NEP)**

Time : 2 Hours

Maximum Marks : 60

**Note :** Answer **all** the Sections.

- Note :**
- (i) Answer **all** the questions from **Section - A**.
  - (ii) Answer **any four** questions from **Section - B**.
  - (iii) Answer **any three** questions from **Section - C**.

**SECTION - A**

1. Answer the following sub-questions. Each sub-question carries **one** mark. **10x1=10**
- (a) Define symmetric matrix.
  - (b) Define rank of the matrix.
  - (c) Define consistency and inconsistency of a system of linear equation.
  - (d) Find the value of  $\lambda$  which the system has non-trivial solution  $7x+4y+3z=0$ ,  $x+2y+\lambda z=0$  and  $x+3y+2z=0$ .
  - (e) Define pedal equation of a polar curve.
  - (f) Write the formula for angle between radius vector and tangent.
  - (g) Evaluate :  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \log(1+x)}$
  - (h) State Cauchy's mean value theorem.
  - (i) Find the 5<sup>th</sup> derivative of  $y=e^{2x}$ .
  - (j) State Leibnitz theorem for the n<sup>th</sup> derivative of a product.

**SECTION - B**

Answer **any four** of the following questions carries **five** marks each.

**4x5=20**

2. Using Cayley-Hamilton's theorem find  $A^{-1}$  if  $A = \begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix}$ .
3. Find the Eigen value and Eigen vector of matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .
4. Show that the curve  $r = a^n \cos n\theta$  and  $r = b^n \sin n\theta$  intersect orthogonally.



5. Evaluate :  $\lim_{x \rightarrow 1} \left[ \frac{x}{x-1} - \frac{1}{\log x} \right]$ .
6. Verify Roll's theorem for the function  $f(x) = x^2 - 4x + 8$  in the interval  $[1, 3]$ .
7. Find the  $n^{\text{th}}$  derivative of  $y = e^{ax} \sin(ax + b)$ .

### SECTION - C

Answer **any three** of the following questions, each question carries **ten** marks.

**3x10=30**

8. (a) Find the rank of the matrix by reducing into normal form. **6**
- $$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$
- (b) If 'A' is a symmetric matrix then show that  $KA$  is also symmetric matrix. **4**
9. (a) Test for consistency and solve **5**
- $$\begin{aligned} x + y + z &= 6 \\ x - y + 2z &= 5 \\ 3x + y + z &= 8 \end{aligned}$$
- (b) Find the non-trivial solution of the system **5**
- $$\begin{aligned} x + 3y - 2z &= 0 \\ 2x - y + 4z &= 0 \\ x - 11y + 14z &= 0 \end{aligned}$$
10. (a) Find the Pedal Equation for  $r^n = a^n \cos n\theta$ . **5**
- (b) Derive the formula for Radius of curvature  $\int = \frac{(1 + y_1^2)^{3/2}}{y_2}$ . **5**
11. (a) Expand  $e^{\sin x}$  using Maclaurin's theorem upto the term containing  $x^4$ . **5**
- (b) Verify the Cauchy's mean value theorem  $f(x) = x^2 + 3$ ,  $g(x) = x^3 + 1$  in  $[1, 3]$ . **5**
12. (a) If  $y = a \cos(\log x) + b \sin(\log x)$  show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ . **5**
- (b) Find the  $n^{\text{th}}$  derivative of  $\sin x \cdot \sin 2x \cdot \sin 3x$ . **5**

