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21BSC3C3MTL

B.Sc. III Semester Degree Examination, April/May - 2024

MATHEMATICS

DSC - III : Ordinary Differential Equations and Real Analysis - I (NEP)

Time : 2 Hours — Maximum Marks : 60

Note : *Answer* ***all*** *sections.*

SECTION - A

1. Answer the following sub-questions, each sub-question carries **one** mark. **10x1=10**

(a) Define degree of differential equation.

(b) Show that $y = a\cos x$ is the solution of the differential equation $\frac{dy}{dx} + y\tan x = 0$.

(c) Find the complementary function of $(D^2 - 5D + 6)y = 0$.

(d) Find the particular integral of $(D^2 + 4)y = \cos 2x$.

(e) Show that the equation $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ is exact.

(f) Write Strum Liouville boundary value problem.

(g) Define total differential equation.

(h) Write the condition for integrability of total differential equation.

(i) Define upper Riemann sum.

(j) Define lower Riemann integral.

SECTION - B

Answer **any four** of the following questions. **4x5=20**

2. Solve : $\frac{dy}{dx} = \frac{1}{\cos(x+y)}$

3. Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = e^{2x} + \cos 2x$

4. Solve : $x\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$, $x > 0$, given that e^x is part of complementary function.

5. Solve : $(yz+2x)dx+(zx-2z)dy+(xy-2y)dz=0$ by verifying the condition of integrability.

6. Show that a constant function is Riemann integrable.

7. Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{-4x} + 5\cos 3x$

SECTION - C

Answer **any three** of the following questions. **3x10=30**

8. (a) Solve : $x^2y\,dx-(x^3+y^3)dy=0$ by choosing integrating factor.

   (b) Find the general and singular solution of $x^2(y-px)=yp^2$ by using the substitution $x^2=u$ and $y^2=v$.

9. (a) Solve : $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

   (b) Sovle the simultaneous equations

$$\frac{dx}{dt} + x = y + e^t$$

$$\frac{dy}{dt} + y = x + e^t$$

10. (a) Solve $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + y = 0$ using the transformation $z=\tan^{-1}x$.

    (b) Sovle $\frac{x^2d^2y}{dx^2} - 2x(x+1)\frac{dy}{dx}+2(x+1)y=x^3 (x>0)$ by changing the dependent variable.

11. (a) Solve $z\,dx + z\,dy + [2(x+y) + \sin z]dz = 0$ by verifying the condition of integrability.

(b) Verify the condition for integrability and solve.

$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$

12. (a) A bounded function $f(x)$ is R-integrable defined on [a, b] if and only if for each $\varepsilon > 0$, f a partition P on [a, b] such that $0 < U(P, f) - L(P, f) < \varepsilon$.

(b) By applying mean value theorem to the integral $\int_0^{\frac{\pi}{4}} \sec x \cdot dx$ show that

$$\frac{\pi}{4} \le \int_0^{\frac{\pi}{4}} \sec x \cdot dx \le \frac{\pi}{2\sqrt{2}}$$

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