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21BSC5C5MTL

B.Sc. V Semester Degree Examination, April/May - 2024

MATHEMATICS

DSC - 5 : Real Analysis - II and Complex Analysis

(NEP)

Time : 2 Hours | Maximum Marks : 60

Note : *Answer **all** sections.*

SECTION - A

1. Answer the following sub-questions, each sub-question carries **one** mark. **10x1=10**

(a) Define segment of the Partition.

(b) Define norm of the partition P.

(c) State first Mean Value theorem.

(d) If $f(x) = \cos x$ find the primitive of $f(x)$.

(e) What is Complex number ?

(f) What is Agrand plane ?

(g) Define transformation.

(h) Define Linear transformation.

(i) If C is made up of $C_1, C_2, C_3, \ldots$ then $\int_C f(z)dz =$

(j) State Green's theorem.

SECTION - B

Answer **any four** of the following questions, each question carries **five** marks. **4x5=20**

2. If x^2 is defined on [0, 1] and $P=\{0, 1/6, 2/6, 3/6, 4/6, 5/6, 1\}$ then find U(p, f) and L(p, f).

3. Evaluate $\int_0^{\pi/4} \left(\sec^4 x - \tan^4 x\right) dx$ by fundamental theorem of integral calculus.

4. Find whether function is differentiable $\sin z$ at i.

5. Prove that Bilinear transformation preserve the cross-ratio of four points.

6. Evaluate $\int_C \left(x^2 - iy^2\right)dz$ along $y=2x^2$ from (1, 2) to (2, 8).

7. Using the substitution $x=\pi-t$ show that $\int_0^{\pi} x\varphi(\sin x)\,dx = \pi/2 \int_0^{\pi} \varphi(\sin x)\,dx$

SECTION - C

Answer **any three** of the following questions, each question carries **ten** marks.

3x10=30

8. (a) State and prove Necessary and Sufficient condition of Riemann integrability.

(b) If $f(x)$ is the function defined on [a, b] by $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$

then find the oscillation of $f(x)$ in [a, b].

9. (a) By applying Mean Value Theorem to the integral $\int_0^{\pi/4} \sec x\,dx$.

Show that $\frac{\pi}{4} \le \int_0^{\pi/4} \sec x\,dx \le \pi/2\sqrt{2}$

(b) Show that $\int_0^{\pi/2} x \cdot \cos x\,dx = \frac{\pi}{2} - 1$ By using integration by parts.

10. (a) Show that $f(z)=\cosh z$ is analytic and hence find $f'(z)$.

(b) Show that $u=e^x\cos y+xy$ is harmonic and find its harmonic conjugate.

11. (a) Find the Bilinear transformation which maps $z_1=-1$, $z_2=0$, $z_3=1$ into $w_1=0$, $w_2=i$, $w_3=3i$.

(b) Find the region in the w-plane bounded by the line $x=1$, $y=1$, $x+y=1$ under the transformation $w=z^2$.

12. (a) If a function $f(z)$ be analytic at all points within and on closed contour C then $\int_C f(z) = 0$.

(b) Evaluate $\int_C \frac{1}{z(z-1)}dz$ where C is the circle $|z|=3$.

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