No. of Printed Pages : **3**

21BSC5C6MTL

B.Sc. V Semester Degree Examination, April/May - 2024

MATHEMATICS

6, DSC-6 : Vector Calculus and Analytical Geometry

(NEP)

Time : 2 Hours | Maximum Marks : 60

Note : *Answer **all** sections.*

SECTION - A

1. Answer the following sub-questions, each sub-question carries **one** mark. **10x1=10**

(a) Prove that if two of three vectors are equal or parallel their scalar triple product vanishes.

(b) Write the Formulae of Serret-Frenet for space curve.

(c) If $\phi = x^2 - y^2$ show that $\nabla^2\phi = 0$

(d) Prove that Curl $(\text{grad}\phi) = 0$

(e) State Stoke's theorem.

(f) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Using Green's theorem.

(g) If (4,2,3) are the direction ratio's of the straight line then find direction cosines of the straight line.

(h) Find the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$.

(i) Write the equation of sphere, its centre and radius.

(j) Define right circular cylinder.

SECTION - B

Answer **any four** of the following questions. Each question carries **five** marks.

4x5=20

2. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.

3. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2, -1, 2)$.

4. Evaluate $\int\int_S \vec{F}.\hat{n}\,ds$ when $\vec{F} = 4xzi - y^2 j + yzk$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$ by divergence theorem.

5. Find the equation of the plane passing through the points $(-1, -2, -3)$, $(3, 4, 5)$, $(0, 6, 2)$.

6. Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$ $(0, 0, 1)$ and $(2, -1, 1)$.

7. Prove that Curl (Curl $\vec{f}$) = grad (div $\vec{f}$) $- \nabla^2 \vec{f}$

SECTION - C

Answer **any three** of the following questions. Each question carries **ten** marks.

3x10=30

8. (a) For the Curve $x=t$, $y=t^2$, $z=t^3$ find the equation of the normal plane at $t=1$.

(b) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ Where 't' is the time.

(i) Find velocity at any time and its magnitude at $t=0$.

(ii) Find acceleration at any time and its magnitude at $t=0$.

9. (a) If $\vec{r} = xi + yj + zk$, $r = \left(\vec{r}\right)$ prove that $\nabla.\left(r^n \vec{r}\right) = (n+3)r^n$.

(b) Find directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2i - j - 2k$.

10. (a) Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = xyi + \left(x^2 + y^2\right)j$ along the path of a straight line from (0, 0) to (1, 0) and then to (1, 1).

(b) Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = 2yi + 3xj - z^2k$ using Stoke's theorem, where 'C' is the boundary of upper half of the surface of the sphere $x^2 + y^2 + z^2 = 9$.

11. (a) Find the equation of the planes bisecting the angle between the planes $3x - 4y + 5z - 3 = 0$ and $5x + 3y - 4z - 9 = 0$.

(b) (i) Find the equation of the line passes through the point $(1, -1, 1)$ and parallel to the vector $i - j + k$.

(ii) Find the equation of the line passing through the point (2, 5, 8) and $(-1, 6, 3)$.

12. (a) Find the condition that the line $\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$ should intersect the polar of the line.

(b) Find the equation of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{72}$.

- o 0 o -