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21BSC2C2MTL

B.Sc. II Semester Degree Examination, Sept./Oct. - 2024

MATHEMATICS

DSC - 2 : Algebra and Calculus II

(NEP)

Time : 2 Hours Maximum Marks : 60

Note : *Answer **all** Parts.*

PART - A

1. Answer **all** questions. **10x1=10**

(a) Define Closed interval.

(b) State Bolzano-Weiestrass theorem.

(c) Define Sub-group.

(d) Every cyclic group is _________.

(e) If $x=r\cos\theta$, $y=r\sin\theta$, show that $\frac{\partial r}{\partial x}=\frac{x}{r}$

(f) Find the degree of homogeneous function $u = \sin^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$.

(g) Evaluate $\int \tan^{-1}(\sin x)\cos x \cdot dx$

(h) Define line integral.

(i) Evaluate $\int_0^1\int_0^2 dx \cdot dy$

(j) Evaluate $\int_0^2\int_0^2\int_0^2 dx \cdot dy \cdot dz$

P.T.O.

PART - B

Answer **any four** of the following. **4x5=20**

2. Determine whether the following sets are bounded or not. Also find their supremum and infimum if exist :

(i) $S = \left\{\frac{1}{n} : n \in N\right\}$

(ii) $S = \left\{(-1)^n \cdot \frac{1}{n} : n \in N\right\}$

3. Show that every cyclic group is abelian.

4. If $u = f(x, y)$ is a homogeneous function of degree n, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.

5. Evaluate $\int_C \left(2y + x^2\right)dx + (3x - y)dy$ along the curve $x = 2t$, $y = t^2 + 3$ where $0 \le t \le 1$.

6. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx \cdot dy \cdot dz$

7. If u and v are functions of two independent variable s and t and s and t themselves are functions of two independent variables x and y, then show that $\frac{\partial(u, v)}{\partial(s, t)} \cdot \frac{\partial(s, t)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(x, y)}$

PART - C

Answer **any three** of the following questions. **3x10=30**

8. (a) Show that supremum (l∪b) of a non-empty set S of real numbers, whenever it exists is unique.

(b) State and prove the Archimedean property of R.

9. (a) State and prove Lagranges theorem.

(b) Show that a non-empty set H of a group G is a sub-group of G if and only if $a, b \in H$ implies $ab^{-1} \in H$.

10. (a) Find the total derivative of u w.r.t 't' when $u = e^x \sin y$ where $x = \log t$ and $y = t^2$.

(b) If $x = r\cos\theta$, $y = r\sin\theta$ find $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$. Also verify $J \cdot J' = 1$

11. (a) Evaluate $\int_C 3x^2 dx + (2xz - y)dy + z \cdot dz$ along the line joining (0, 0, 0) and (2, 1, 3).

(b) Evaluate $\int_C \left(\frac{a^2 y^2}{b^2} + \frac{b^2 x^2}{a^2} \right) ds$ around the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

12. (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \int_0^{\sqrt{a^2 - x^2 - y^2}} \frac{dx\,dy\,dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$

(b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$

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