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21BSC6C13MAL

B.Sc. VI Semester Degree Examination, Sept./Oct. - 2024

MATHEMATICS

DSC - 7 : Linear Algebra

(NEP)

Time : 2 Hours Maximum Marks : 60

Note : *Answer **all** sections.*

SECTION - A

1. Answer the following sub-questions, each sub-question carries **one** mark. **10x1=10**

(a) Define Vector Subspaces.

(b) Show that the set $S=\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is Linearly independent in $V_3(R)$.

(c) If $T : V_1(R) \to V_3(R)$ is defined by $T(x)=(x, x^2, x^3)$, verify whether T is Linear or not.

(d) State Rank-Nullity Theorem.

(e) Define homomorphism of a Vector Space.

(f) Define Diagonalization of a Matrix.

(g) What are the conditions for invertibility ?

(h) Define Minimal polynomial of a transformation.

(i) What is inner product in a Vector Spaces ?

(j) Determine if the Vectors $u=(1, 2, -1)$ and $v=(2, -1, 1)$ are Orthogonal.

SECTION - B

Answer **any four** of the following questions, each question carries **five** marks. **4x5=20**

2. Express the Vectors $(2, -1, -8)$ as a linear combination of the Vectors $(1, 2, 1)$, $(1, 1, -1)$ and $(4, 5, -2)$.

3. Define Basis and Dimension of V(F) determine whether the set $\{(1, 2, 3,), (-2, 1, 3), (3, 1, 0)\}$ is a basis of $V_3(R)$.

4. Find the algebraic and geometric multiplicities of Eigenvalues of a linear transformation $A = \begin{pmatrix} 4 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

5. Determine the linear transformation $T : R^2 \to R^2$ defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x+y \\ x-y \end{bmatrix}$. Determine whether T is invertible or not. If so, find the inverse of T.

P.T.O.

6. Show that the function $< -, - > : R^3 \times R^3 \to R$ defined by $<u, v> = \sum_{i=1}^{3} uivi$, where $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are Vectors in R^3 is an inner product.

7. Diagonalize the linear transformation $T : R^2 \to R^2$ represented by the Matrix $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$.

SECTION - C

Answer **any three** of the following questions, each question carries **ten** marks.
3x10=30

8. (a) The union of two subspaces of a Vector Space V over a field F is a subspace off one is contained in the other.

(b) Prove that (3, −7, 6) is the span of the Vectors (1, −3, 2), (2, 4, 1) and (1, 1, 1).

9. (a) Let $T : V \to W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find Range, null space, Rank, nullity and hence verify Rank-Nullity Theorem.

(b) Find the Matrix of Linear Transformation $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (2y - x, y, 3y - 3x)$ relative basis $B_1 = \{(1, 1), (-1, 1)\}$ and $B_2 = \{(1, 1, 1), (1, -1, 1)\ (0, 0, 1)\}$.

10. (a) Find Eigen values and Eigen vectors of a linear transformation $T : R^2 \to R^2$ defined by $A(v) = Av$, where $A = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) Find the Eigen space of the linear transformation $T : R^2 \to R^2$ represented by the Matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$.

11. (a) Consider the linear transformation $T : R^2 \to R^2$ defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{pmatrix} x + 2y \\ 2x + 4y \end{pmatrix}$ Determine whether T is singular or not.

(b) Determine the relationship between the minimal and characterstic polynomials of the linear transformation of $T : R^2 \to R^2$ defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x - y \\ 2x \end{bmatrix}$.

12. (a) State and prove Cauchy-Schwarz inequality.

(b) Define Orthonormal Basis and determine if the Set of Vectors $u = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ $v = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ and $w = (0, 0, 1)$ forms an orthonormal basis for R^3.

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