21BSC4C4MTL

B.Sc. IV Semester Degree Examination, Sept./Oct. - 2024
MATHEMATICS

DSC - 4 : Integral Transform and Partial Differential Equations
(NEP)

Time : 2 Hours   Maximum Marks : 60

Note : *Answer **all** Sections.*

SECTION - A

Answer the following sub-questions, each sub-question carries **one** mark.   **10x1=10**

1. (a) What is Laplace transform of t ?

(b) What is inverse Laplace transform of $\left(\dfrac{1}{s+4} \right)$?

(c) If $f(x) = x^3$ find the Fourier co-efficient of a_0 in $(-\pi, \pi)$.

(d) Define periodic function.

(e) Define Fourier sine transform.

(f) Write inverse formula for Fourier transform.

(g) What is the z-transform of n ?

(h) What is the inverse z-transform of $\left[\dfrac{z^2+z}{(z-1)^3} \right]$?

(i) Give an example of partial differential equation.

(j) A linear partial differential equation of the form is $P_p + Q_q = R$ is called _____.

SECTION - B

Answer **any four** of the following questions.   **4x5=20**

2. Verify the Convolution theorem for the function $f(t) = 1$, $g(t) = \sin t$ by applying Laplace transform.

3. Obtain Fourier series of $f(x) = e^{-ax}$ in $-\pi < x < \pi$.

4. Find the Fourier transform of $f(x) = e^{-|x|}$.

5. Obtain the z-transform of $\cos n\theta$.

6. Solve $p\tan x + q\tan y = \tan z$.

7. Obtain the Fourier series expansion of the function $f(x) = \begin{cases} x \text{ in } 0 < x < \pi \\ x - 2\pi \text{ in } \pi < x < 2\pi \end{cases}$

Hence deduce that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} + \ldots\ldots$

SECTION - C

Answer **any three** of the following questions.   **3x10=30**

8. (a) Find the inverse Laplace transform of the function $\dfrac{3s^2 + 16s + 26}{s\left(s^2 + 4s + 13\right)}$

   (b) Apply Laplace transform to solve $\dfrac{dx}{dt} = 2x - 3y$; $\dfrac{dy}{dt} = y - 2x$ given $x(0) = 8$ $y(0) = 3$

9. (a) Find the Fourier expansion for the function defined by $f(x) = \begin{cases} -1 \text{ in } -3 < x < 0 \\ 0 \text{ in } x = 0 \\ 1 \text{ in } 0 < x < 3 \end{cases}$

   (b) Obtain half range sine series of function $f(x) = x^2$ in $0 < x < \pi$

10. (a) Find the Fourier cosine transform of the function $f(x) = \begin{cases} x \ 0 < x < a \\ 0 \text{ otherwise} \end{cases}$

   (b) Modulation theorem : If $F(x)$ has the Fourier transform $f(s)$ then prove that $F(x) \cos(ax)$ has the Fourier transform $\dfrac{1}{2}\left[f(s-a) + f(s+a)\right]$

11. (a) Given $Z_T(u_n) = \dfrac{2z^2 + 3z + 4}{(z-3)^3}$, $|z| > 3$ show that $u_1 = 2$, $u_2 = 21$

(b) Solve the difference equation $y_{n+2} + y_n = 0$ by using z-transform.

12. (a) Find the complete integral of $px + qy = pq$ by Charpits method.

(b) Solve : $z^2(p^2 x^2 + q^2) = 1$

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