No. of Printed Pages : **3**

21BSC5C5MTL

B.Sc. V Semester Degree Examination, Sept./Oct. - 2024

MATHEMATICS

DSC-5 : Real Analysis-II & amp; Complex Analysis

(NEP)

Time : 2 Hours | Maximum Marks : 60

Note : *Answer **all** sections.*

SECTION - A

Answer the following sub-questions, each sub-question carries **one** mark. **10x1=10**

1. (a) Define closed interval and give example.

(b) Let [a, b]=[2, 8] and P={2, 4, 6, 8} and sub interval [2, 4], [4, 6], [6, 8] find the norm.

(c) State Second Mean Value Theorem.

(d) Complete state if $f(x)$ and $g(x)$ are differentiable function on [a, b] and $f'(x)$ and $g'(x)$ are continuous on [a, b] then $\int_a^b f(x).g'(x)\,dx =$ ________.

(e) Write complex number in polar form.

(f) What is complex variable ?

(g) Define open set.

(h) Define Jacobian of a transformation.

(i) What is Contour ?

(j) What is Simple Curve ?

SECTION - B

Answer **any four** of the following questions. **4x5=20**

2. Prove that if $f(x)$ is a real valued bounded function defined on [a, b] and $p\epsilon\phi[a, b]$ then $m(b-a)\leq L(p, f)\leq U(p, f)\leq M(b-a)$ where M and m are respectively the superimum and infimum of $f(x)$ on [a, b].

3. Can we evaluate $\int_{-1}^{1}\frac{dx}{1+x^2}$ by substituting $x=\frac{1}{t}$?

4. Show that $\omega=z+e^{z}$ is analytic and hence find $\frac{d\omega}{dz}$.

5. Prove that Bilinear transformation preserve the cross-ratio of four points.

6. Evaluate $\int_{C}z^2dz$ where C is the line join point 0 and $3+i$.

7. By applying Mean Value Theorem to the integral $\int_{0}^{1/4}\frac{dx}{\sqrt{1-x^2}}$ show that

$$\frac{1}{4}\leq\int_{0}^{1/4}\frac{dx}{\sqrt{1-x^2}}\leq\frac{1}{\sqrt{15}}.$$

SECTION - C

Answer **any three** of the following questions. **3x10=30**

8. (a) Show that the function $f(x)=x^2$ is integrable on [0, a] and $\int_{0}^{a}x^2dx=\frac{a^3}{3}$.

(b) If $f(x)=2x-1$ $0\leq x\leq 1$ and $P=\left\{0,\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7},1\right\}$ find L(p, f).

9. (a) Show that $\int_0^1 \frac{1+x}{(2+x)^2}\,dx = \frac{e}{3} - \frac{1}{2}$ by integration by part Method.

(b) Prove that if $f(x) \epsilon P[a, b]$ and $\phi(x)$ is a primitive of $f(x)$ then $\int_a^b f(x)\,dx = \phi(b) - \phi(a)$

10. (a) Find the analytic function whose real part is $x^3 - 3xy^2$.

(b) Evaluate $\lim_{z \to e^{i\pi/4}} \frac{z^2}{z^4 + z + 1}$.

11. (a) Find the image of the circle $|z| = 1$ and $|z| = 2$ [equivalently $x^2 + y^2 = 1$; $x^2 + y^2 = 4$] under the mapping $\omega = z + (1/z)$.

(b) Find the bilinear transformation which map the points $z = 1, i, -1$ into $\omega = i, 0, -i$.

12. (a) Evaluate $\int_C \frac{z}{(z^2+1)(z^2-9)}$ where C is the circle $|z| = 2$

(b) Prove that if $f(z)$ is analytic within and on a simple closed curve c and z=a is a point within C then $f'(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^2}\,dz$.

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