No. of Printed Pages : **3**

21BSC5C6MTL

B.Sc. V Semester Degree Examination, Sept./Oct. - 2024

MATHEMATICS

DSC-6 : Vector Calculus and Analytical Geometry

(NEP)

Time : 2 Hours | Maximum Marks : 60

Note : *Answer **all** sections.*

SECTION - A

1. Answer the following sub-questions, each sub-question carries **one** mark. **10x1=10**

(a) Find the volume of parallelopiped whose co-terminous edges are $2i-3j+k$, $i-j+2k$, $2i+j-k$.

(b) Prove that, if two of three vectors are equal or parallel their scalar triple product vanishes.

(c) Find div $\vec{F}$ if $\vec{F} = 3x^2 i + 5xy^2 j + xyz^3 k$ at (1, 2, 3).

(d) Prove that $\text{div}\left(\text{curl}\ \vec{f}\right) = 0$.

(e) State Stoke's theorem.

(f) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem.

(g) Find the angle between the planes $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$

(h) Write the equation of the plane passing through the point (x_1, y_1, z_1).

(i) Define right circular cylinder.

(j) Write the standard equation of the sphere, its centre and radius.

P.T.O.

SECTION - B

Answer **any four** of the following questions. Each question carries **five** marks. **4x5=20**

2. Find the value of P. Show that the vectors $2i-j+k$, $i+2j-3k$ and $3i+pj+5k$ are coplanar.

3. If $\vec{r} = xi + yj + zk$ such that $|\vec{r}| = r$ prove that $\nabla f(r) = \left(\frac{f^1(r)}{r}\right)\vec{r}$

4. Evaluate $\int_c \vec{F}\cdot d\vec{r}$ where $\vec{F} = xyi + (x^2+y^2)j$ along the path of a straight line from (0, 0) to (1, 0) and then to (1, 1).

5. Find the equation of the plane passing through the point $3i+3j+4k$ and perpendicular to the vector $12i-4j+3k$. Reduce the equation to the normal form.

6. Find the equation of the sphere which passes through the point (1, 0, 0), (0, 1, 0), (0, 0, 1) and its centre on the plane $x+y+z=6$.

7. $\text{Curl}\left(\text{curl}\,\vec{f}\right) = \text{grad}\left(\text{div}\,\vec{f}\right) - \nabla^2\vec{f}$

SECTION - C

Answer **any three** of the following questions. Each question carries **ten** marks. **3x10=30**

8. (a) Show that $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$

(b) For the curve $x=t$, $y=t^2$, $z=t^3$ find the equation of the normal plane at $t=1$.

9. (a) Find the directional derivative of $\oint = \frac{xz}{x^2+y^2}$ at (1, −1, 1) in the direction of $\vec{A} = i - 2j + k$

(b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, −1, 2).

10. (a) Verify Green's theorem $\oint_c \left(3x^2 - 8y^2\right) dx + (4y - 6xy)dy$ where c is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

(b) Using Gauss divergence theorem Evaluate $\iint_s \left(xi + yj + z^2k\right)\vec{n}\, ds$, where s is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $Z = 1$.

11. (a) Find the equation of the plane which passing through the points (2, 2, − 1) and parallel to the line joining the points A(3, − 1,0), B(2, 1, 0) and C(1, − 1,0), D(− 1, 2, 0).

(b) Find the equation of the line passing through the point (2, 5, 8) and (− 1, 6, 3).

12. (a) Find the equation of the tangent plane to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{72}$

(b) The radius of the normal section of the right circular cylinder is 2 units and the axis lies along the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$ find its equation.

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